Cambridge Assessment International Education

Cambridge International AS & A Level

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

1 Solve the equation $3x + 2 = \dfrac{2}{x-1}$.

$(3x+2)(x-1) = 2$

$3x^2 - 3x + 2x - 2 = 2$

$3x^2 - x - 2 - 2 = 0$

$3x^2 - x - 4 = 0$

$(3x-4)(x+1) = 0$

$\boxed{x = 4/3 , \quad x = -1}$

2 The equation of a curve is such that $\frac{dy}{dx} = 12(\frac{1}{2}x - 1)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

at $P(6,4)$ $\quad \frac{dy}{dx} = 12\left(\frac{1}{2}(6) - 1\right)^{-4}$

$\frac{dy}{dx} = 3/4$

Equation of tangent at $P(6,4)$, $m = 3/4$

$y - y_1 = m(x - x_1)$

$y - 4 = 3/4\,(x - 6)$

$\boxed{y = \frac{3}{4}x - \frac{1}{2}}$

(b) Find the equation of the curve. [4]

$\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$

$\int dy = \int 12(\tfrac{1}{2}x - 1)^{-4} \cdot dx$

$y = \dfrac{12\left(\frac{1}{2}x - 1\right)^{-3}}{-3 \times \frac{1}{2}} + c$

$y = -8\left(\frac{1}{2}x - 1\right)^{-3} + c$

$x = 6, y = 4$ $\quad 4 = -8\left(\frac{1}{2}x - 1\right)^{-3} + c$

$4 = -1 + c$

$\boxed{c = 5}$

$\boxed{y = -8\left(\frac{1}{2}x - 1\right)^{-3} + 5}$

[Turn over

3 A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary po[int] at the point P, which has x-coordinate 9.

Find the y-coordinate of P.

$x = 9$, $\frac{dy}{dx} = 0$

$y = ax^{1/2} - 2x$

$\frac{dy}{dx} = \frac{1}{2} \times a \times x^{-1/2} - 2$

$\frac{dy}{dx} = a \times \frac{1}{2} \times 9^{-1/2} - 2$

$0 = \frac{1}{6}a - 2$

$\boxed{a = 12}$

$y = 12x^{1/2} - 2x$

at $x = 9$, $y = 12 \times \sqrt{9} - 2 \times 9$

$y = 36 - 18$

$\boxed{y = 18}$

4 The coefficient of x^2 in the expansion of $\left(1 + \frac{2}{p}x\right)^5 + (1 + px)^6$ is 70.

Find the possible values of the constant p. $\boxed{(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^r b^{n-r}}$ [6]

$\left(1 + \frac{2}{p}x\right)^5 + (1+px)^6$

$\left(\frac{2}{p}x+1\right)^5 + (px+1)^6.$

Term in x^2

${}^5C_2 1^3 \left(\frac{2x}{p}\right)^2$

$10 \times \frac{4x^2}{p^2}$

$\left(\frac{40}{p^2}\right)x^2$

Term in x^2

${}^6C_2 (1)^4 (px)^2$

$(15p^2)x^2$

Coefficient of $x^2 = 70$

$15p^2 + \frac{40}{p^2} = 70$

$15p^4 + 40 = 70p^2$

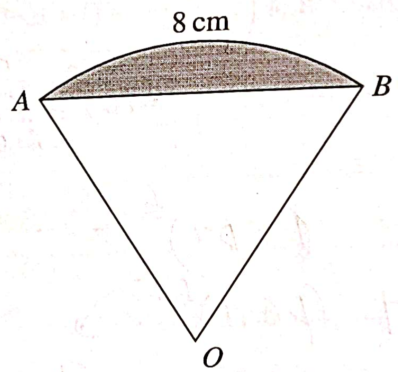
$15p^4 - 70p^2 + 40 = 0 \quad \Rightarrow 3p^4 - 12p^2 - 2p^2 + 8 = 0.$

$(p^2-4)(3p^2-2) = 0.$

$(p+2)(p-2)(3p^2-2) = 0.$

$\boxed{p = \pm 2, \ \pm\sqrt{2/3}}$

5



The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

let $OA = OB = r$, Angle $AOB = \theta$

$\boxed{r\theta = 8}$ — 1

$r + r + r\theta = 20$

$2r + 8 = 20$

$2r = 12$

$\boxed{r = 6}$ — ii

$r\theta = 8$

$6\theta = 8$

$\theta = 8/6$

$\boxed{\theta = 4/3 \text{ rad}}$

$AB = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6 \times \cos(4/3)}$

$AB = \sqrt{72 - 72\cos 4/3}$

Perimeter = $\overline{AB}$ + arc AB

$= \sqrt{72 - 72\cos 4/3} + 8$

$\boxed{15.4}$ cm

(b) Find the area of the shaded segment. [2]

Area of shaded Segment =

Area of Sector OAB − Area of triangle OAB

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$\frac{1}{2}\times 6^2 \times \frac{4}{3} - \frac{1}{2}\times 6^2 \times \sin 4/3$

$\boxed{6.51\ cm^2}$

6 **(a)** Show that the equation

$$\frac{1}{\sin\theta+\cos\theta}+\frac{1}{\sin\theta-\cos\theta}=1$$

may be expressed in the form $a\sin^2\theta+b\sin\theta+c=0$, where a, b and c are constants to be found. [3]

$$\frac{\sin\theta-\cos\theta+\sin\theta+\cos\theta}{(\sin\theta+\cos\theta)(\sin\theta-\cos^2\theta)}=1$$

$$\frac{2\sin\theta}{\sin^2\theta-\cos^2\theta}=1$$

$$2\sin\theta=\sin^2\theta-\cos^2\theta.$$

$$\sin^2\theta-\cos^2\theta-2\sin\theta=0.$$

$$\sin^2\theta-(1-\sin^2\theta)-2\sin\theta=0.$$

$$\sin^2\theta-1+\sin^2\theta-2\sin\theta=0.$$

$$\boxed{2\sin^2\theta-2\sin\theta-1=0}$$

$$\boxed{a=2,\quad b=-2,\quad c=-1}$$

(b) Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

$2\sin^2\theta - 2\sin\theta - 1 = 0$

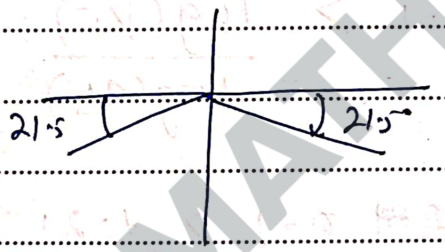
$a = 2, \quad b = -2, \quad c = -1$

$\sin\theta = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4\times 2\times(-1)}}{2\times 2}$

$\sin\theta = \dfrac{2 \pm \sqrt{12}}{4}$

$\sin\theta = -0.3660$ $\qquad$ $\sin\theta = 1.3660$

$\theta = \sin^{-1}(-0.3660)$ $\qquad$ (Not possible).



$$\boxed{\begin{aligned} \theta &= 180 + 21.5 = 201.5° \\ \theta &= 360 - 21.5 = 338.5° \end{aligned}}$$

7 A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

G.P : 50, 40, 32

$a = 50$

$r = 40/50 = 4/5$

$T_n < 10$ $\quad a \times r^{n-1} < 10$

$50\left(4/5\right)^{n-1} < 10$

$\left(4/5\right)^{n-1} < 1/5$

$\log\left(4/5\right)^{n-1} < \log\left(1/5\right)$

$(n-1)\log\left(4/5\right) < \log\left(1/5\right)$

$n-1 > \dfrac{\log(1/5)}{\log(4/5)}$

$n-1 > 7.21$

$n > 8.21$

$\boxed{\text{minimum } n = 9}$ ✓ 9th impact is the first one in which post sinks less than 10 mm.

9th impact = 9th term

$= ar^8$

$= 50\left(4/5\right)^8$

$8.388608 < 10$

✓

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{20} = \frac{50\left(1-(4/5)^{20}\right)}{1-4/5}$$

$$S_{20} = 247.11269 \simeq 247\ mm$$

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{50}{1-4/5}$$

Greatest total depth = 250

8 The function f is defined by $f(x) = 2 - \dfrac{3}{4x - p}$ for $x > \dfrac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

$$f(x) = 2 - \frac{3}{4x-p} = 2 - 3(4x-p)^{-1}$$

$$f'(x) = -3(-1)(4x-p)^{-2}(4)$$

$$= \frac{-12}{(4x-p)^2}$$

Given, ~~x<1/2~~

For all values of x, $(4x-p)^2 > 0$.

$f'(x) > 0$,

Hence f is an increasing function.

Or

$x > p/4$

$4x > p$

$4x - p > 0$

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx - d}$, where a, b, c and d are integers. [4]

$$f(x) = 2 - \frac{3}{4x-p}$$

let $y = f(x) \Rightarrow y = 2 - \frac{3}{4x-p}$.

$$\frac{3}{4x-p} = 2-y$$

$$\frac{3}{2-y} = 4x-p.$$

$$4x = \frac{3}{2-y} + p.$$

$$x = \frac{1}{4}\left[\frac{3}{2-y} + p\right]$$

$$f^{-1}(x) = \frac{3}{8-4x} + \frac{p}{4}.$$

$$= \frac{p}{4} - \frac{3}{-8+4x}$$

$$\frac{p}{4} - \frac{3}{4x-8}$$

$a = 4$, $b = 3$, $c = 4$, $d = 8$

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

$$f^{-1}(x) \equiv f(x)$$

$$\frac{p}{4} - \frac{3}{4x-8} = 2 - \frac{3}{4x-p}$$

Comparing the Coefficient.

$\boxed{p = 8}$

9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x-a)^2 + b$. [1]

$f(x) = x^2 - 4x + 9$
$= (x-2)^2 - 2^2 + 9$
$= (x-2)^2 - 4 + 9$
$f(x) = (x-2)^2 + 5$

(b) Express $g(x)$ in the form $2[(x+c)^2 + d]$. [2]

$g(x) = 2x^2 + 4x + 12$
$= 2[x^2 + 2x + 6]$
$= 2[(x+1)^2 - 1^2 + 6]$
$= 2[(x+1)^2 - 1 + 6]$

$\boxed{g(x) = 2[(x+1)^2 + 5]}$

(c) Express $g(x)$ in the form $kf(x+h)$, where k and h are integers. [1]

$f(x) = (x-2)^2 + 5$ $\Longrightarrow$ $f(x+3) = (x+3-2)^2 + 5$

$g(x) = 2[(x+1)^2 + 5]$ $f(x+3) = (x+1)^2 + 5$

$\boxed{g(x) = 2f(x+3)}$

$k = 2$

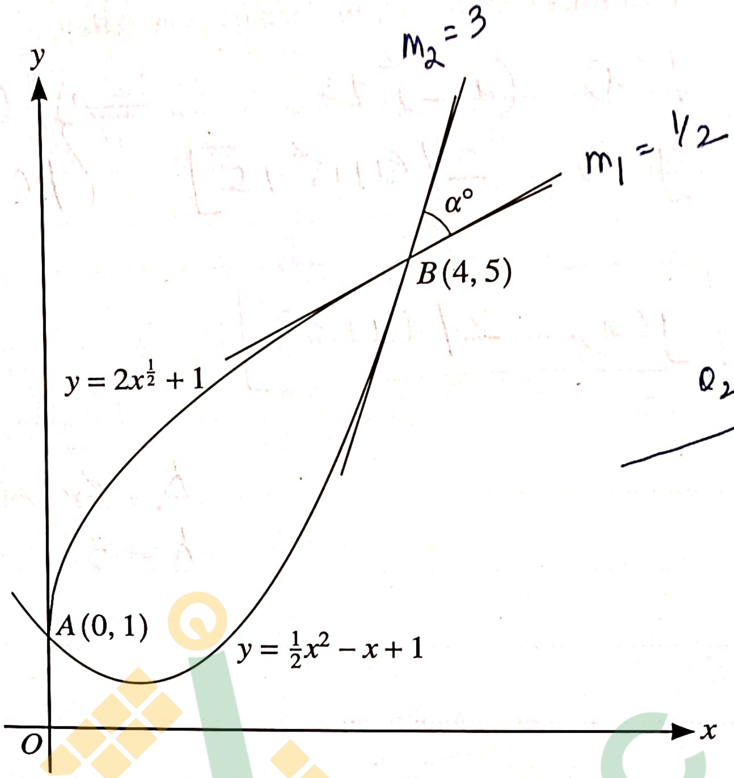
$h = 3$

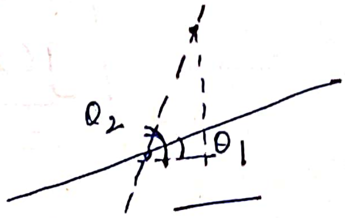
(d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

Translation by vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ $f(x+3)$

Stretch by factor 2 parallel to y-axis. $2f(x+3)$

10





Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

Area enclosed = $\int_0^4 \left[(2x^{1/2}+1) - (\frac{1}{2}x^2 - x + 1)\right] dx$

$\left[\frac{2x^{3/2}}{3/2} + \cancel{x} - \frac{1}{2}\frac{x^3}{3} + \frac{x^2}{2} \cancel{-x}\right]_0^4$

$\left[\frac{4}{3}x^{3/2} - x^3/6 + x^2/2\right]_0^4$

$\left[\frac{4}{3} \times 4^{3/2} - 4^3/6 + 4^2/2\right] - 0$

$32/3 - 32/3 + 8$

$\boxed{8}$

The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

(b) Find α. [5]

$y_1 = 2x^{1/2} + 1$

$\frac{dy_1}{dx} = 2 \times \frac{1}{2} \times x^{-1/2} = x^{-1/2}$

at $B(4,5)$ $\frac{dy_1}{dx} = (4)^{-1/2} = \boxed{m_1 = 1/2}$

let $y_2 = \frac{1}{2}x^2 - x + 1$

$\frac{dy_2}{dx} = \frac{1}{2} \times 2 \times x - 1 = x - 1$

at $B(4,5)$ $\frac{dy_2}{dx} = 4 - 1 = \boxed{m_2 = 3}$

$m_1 = \tan \theta_1$ $\qquad m_2 = \tan \theta_2 \Rightarrow \theta_2 = \tan^{-1}(3)$

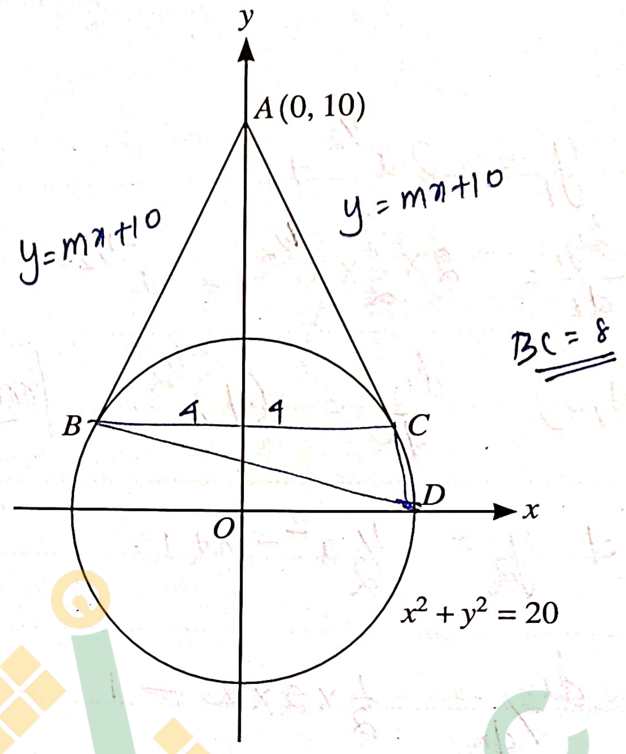
$\theta_1 = \tan^{-1}(1/2)$

$\alpha = \theta_2 - \theta_1$

$\alpha = \tan^{-1}(3) - \tan^{-1}(1/2)$

$\boxed{\alpha = 45°}$

11



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point A (0, 10).

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

$x^2+y^2=20$ —— (i)

$y = mx+10$ —— (ii)

$x^2+(mx+10)^2 = 20$

$x^2+m^2x^2+20mx+100=20$

$x^2+m^2x^2+20mx+80=0$

$x^2(1+m^2)+20mx+80=0$

$a=(1+m^2)$ $b=20m$, $c=80$.

for tangency $b^2-4ac=0$

$(20m)^2 - 4\times(1+m^2)\times(80)=0$

$80m^2-320=0$

$m=\sqrt{\frac{320}{80}}$

$\boxed{m=\pm 2}$

(b) Find the coordinates of B and C. [3]

$BA: \quad y = 2x + 10$

$CA: \quad y = -2x + 10$

from: $x^2(1+m^2) + 20mx + 80 = 0$

for $m = 2$, $x^2(1+2^2) + 20(2)x + 80 = 0$

$5x^2 + 40x + 16 = 0$

$5(x+4)^2 = 0$

$\boxed{x = -4}$

For $x = -4$

$y = 2(-4) + 10$

$y = 2$

$\boxed{B\ (-4, 2)}$

for $m = -2$, $x^2(1 + (-2)^2) + 20(-2)x + 80 = 0$

$5x^2 - 40x + 80 = 0$

$5(x^2 - 8x + 16) = 0$

$5(x-4)^2 = 0 \quad \boxed{x = 4}$

for $x = 4$

$y = -2(4) + 10$

$y = 2$

$\boxed{C\ (4, 2)}$

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]

$x^2 + y^2 = 20$

at D, $y = 0$, $x^2 = 20$

$x = \pm\sqrt{20}$

$x = \pm 2\sqrt{5}$

$D\ (2\sqrt{5}, 0)$

$B\ (-4, 2)$

$C\ (4, 2)$

$BD = \sqrt{(2\sqrt{5} - (-4))^2 + (0-2)^2} = 8.705$

$CD = \sqrt{(2\sqrt{5} - 4)^2 + (0-2)^2} = 2.05497$

Angle $B\hat{D}C = \cos^{-1}\left[\dfrac{8.705^2 + 2.054^2 - 8^2}{2 \times 8.705 \times 2.05}\right]$

$\boxed{\text{Angle } B\hat{D}C = 63.4^\circ}$