1 A summary of 50 values of x gives

$$\Sigma(x-q) = 700, \qquad \Sigma(x-q)^2 = 14\,235,$$

where q is a constant.

(a) Find the standard deviation of these values of x. [2]

Standard deviation is not affected by coding

$$\sigma = \sqrt{\frac{\Sigma(x-q)^2}{n} - \left[\frac{\Sigma(x-q)}{n}\right]^2} \quad \text{or} \quad \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{14235}{50} - \left(\frac{700}{50}\right)^2}$$

$$\sigma = 9.4180$$

$$\sigma = 9.42 \text{ (3 sf)}$$

(b) Given that $\Sigma x = 2865$, find the value of q. [2]

$$\Sigma(x-q) = \Sigma x - nq$$

$$700 = 2865 - 50q$$

$$50q = 2865 - 700$$

$$q = \frac{2165}{50}$$

$$q = 43.3$$

2 Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. [3]

$P(X=1) = P(HTTT) + P(THTT) + P(TTHT) + P(TTTH)$

One biased coin

$\rightarrow P(H) = 0.6$

$P(T) = 0.4$

$= 0.6 \times (0.5)^3 + 0.4 \times (0.5)^3 + 0.4 \times (0.5)^3 + 0.4 \times (0.5)^3$

$= 0.075 + 0.4 \times (0.5)^3 \times 3$

$= 0.075 + 0.15$

Other three fair coin

$= 0.225$

$\rightarrow P(H) = 0.5$

$P(T) = 0.5$

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	0.05	0.225	0.375	0.275	0.075

$P(X=3) = P(HHHT) + P(HHTH) + P(HTHH) + P(THHH)$

$= 0.6 \times (0.5)^3 \times 3 + 0.4 \times (0.5)^3$

$= 0.275$

$P(X=2) = 1 - (0.05 + 0.225 + 0.275 + 0.075)$

$= 0.375$

(c) Given that E(X) = 2.1, find the value of Var(X). [2]

x^2	0	1	4	9	16	
x	0	1	2	3	4	
P($X = x$)	0.05	0.225	0.375	0.275	0.075	
$x \cdot p$	0	0.225	0.75	0.825	0.3	$= \boxed{2.1}$ Σxp
$x^2 \cdot p$	0	0.225	1.5	2.475	1.2	$= \boxed{5.4}$ $\Sigma x^2 p$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= 5.4 - (2.1)^2$$

$$= 0.99$$

3.

A red spinner has four sides labelled 1, 2, 3, 4. When the spinner is spun, the score is the number on the side on which it lands. The random variable X denotes this score. The probability distribution table for X is given below.

x	1	2	3	4
$P(X = x)$	0.28	p	$2p$	$3p$

(a) Show that $p = 0.12$. [1]

$0.28 + p + 2p + 3p = 1$

$6p = 1 - 0.28$

$p = \frac{0.72}{6}$

$p = 0.12$

A fair blue spinner and a fair green spinner each have four sides labelled 1, 2, 3, 4. All three spinners (red, blue and green) are spun at the same time.

(b) Find the probability that the sum of the three scores is 4 or less. [3]

For Red Spinner $P(1) = 0.28$ $P(2) = 0.12$

For Blue and Green Spinner ; $P(1) = \frac{1}{4}$ $P(2) = \frac{1}{4}$

Fair Spinners

Possible Outcomes (Sum should be 4 or less)

R	B	G
1	1	2
1	2	1
1	1	1
2	1	1

$P(S \le 4) = P(1,1,2) + P(1,2,1) + P(1,1,1) + P(2,1,1)$

$= 0.28 \times \frac{1}{4} \times \frac{1}{4} + 0.28 \times \frac{1}{4} \times \frac{1}{4} + 0.28 \times \frac{1}{4} \times \frac{1}{4} + 0.12 \times \frac{1}{4} \times \frac{1}{4}$

to denote sum of three score

$= 0.0175 + 0.0175 + 0.0175 + 0.0075$

$= 0.06$

Conditional Probability

(c) Find the probability that the product of the three scores is 4 or less given that X is odd. [5]

Let Y represent product of the 3 scores that is less or equal to 4.

R	B	G	
1	1	1	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	1	2	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	1	3	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	1	4	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	2	1	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	2	2	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	3	1	$0.28 \times 0.25 \times 0.25 = 0.0175$
1	4	1	$0.28 \times 0.25 \times 0.25 = 0.0175$
3	1	1	$\underbrace{2(0.12)}_{2P} \times 0.25 \times 0.25 = 0.015$

$$P(X) = P(X=1) + P(X=3)$$

$$= 0.28 + 2(0.12)$$

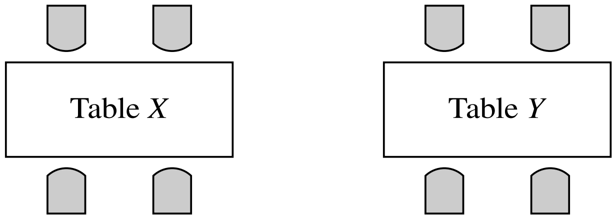
$$= 0.52$$

$$P(Y/X) = \frac{P(Y \cap X)}{P(X)}$$

$$= \frac{(0.175 \times 8) + 0.015}{0.52}$$

$$= \frac{31}{104} = 0.298$$

4.



In a restaurant, the tables are rectangular. Each table seats four people: two along each of the longer sides of the table (see diagram). Eight friends have booked two tables, X and Y. Rajid, Sue and Tan are three of these friends.

(a) The eight friends will be divided into two groups of 4, one group for table X and one group for table Y.

Find the number of ways in which this can be done if Rajid and Sue must sit at the same table as each other and Tan must sit at the other table. [3]

R S, T, x, y or T, R S, x, y

8 persons < R S T, 5 Persons

No of ways: $({}^5C_2 \times {}^3C_3) + ({}^5C_3 \times {}^2C_2)$

$10 + 10$

20

When the friends arrive at the restaurant, Rajid and Sue now decide to sit at table X on the same side as each other. Tan decides that he does not mind at which table he sits.

(b) Find the number of different seating arrangements for the 8 friends. [3]

R S, x, T, y or x, R S, T, y

No of arrangements =

$2! \times 6! \times 2$

R & S can sit on either side of table

R & S can sit in 2! ways along side each other

No of ways other friends can occupy the remaining seats

$= 2880$

Altenatively

2 people chosen sit in 2! ways

R & S can sit on either side of table x

No. of arrangements = ${}^6C_2 \times 2! \times 2! \times 2$

2! R S

x

y

${}^6C_2$

2 people chosen from 6 to fill table x

R & S sit in 2! ways alongside each other

= 2880

As they leave the restaurant, the 8 friends stand in a line for a photograph.

(c) Find the number of different arrangements if Rajid and Sue stand next to each other, but neither is at an end of the line. [4]

(R & S together) _ _ R S _ _ _ _ $7! \times 2!$

R & S together and at the end of a line

R S _ _ _ _ _ _ $6!$

S R _ _ _ _ _ _ $6!$

_ _ _ _ _ _ R S $6!$

_ _ _ _ _ _ S R $6!$

No of arrangements = No of Arrangement [R & S together] − No of arrange [R & S together at the end of line]

$7! \times 2! - 6! \times 4$

$10080 - 2880$

7200

5.

Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

3 Red   4 Orange   5 Yellow

P(Same Colour) = P(3 red) + P(3 orange) + P(3 Yellow)

$$\frac{{}^{3}C_3}{{}^{12}C_3} + \frac{{}^{4}C_3}{{}^{12}C_3} + \frac{{}^{5}C_3}{{}^{12}C_3}$$

$$\frac{1}{220} + \frac{4}{220} + \frac{10}{220}$$

$$\frac{15}{220} = \frac{3}{44}$$

Alternatively: or

P(RRR) + P(OOO) + P(YYY)

$$\left(\frac{3}{12} \times \frac{2}{11} \times \frac{1}{10}\right) + \left(\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}\right) + \left(\frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}\right)$$

$$\frac{90}{1320}$$

$$= \frac{3}{44}$$

$$P(YYY) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} = \frac{60}{1320} = \frac{10}{220}$$

conditional probability

(b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

$P(\text{yellow} \mid \text{Same colour Sweet})$

3y, 3R, 3O

$= \dfrac{P(\text{yellow} \cap \text{Same colour Sweet})}{P(\text{Same colour Sweet})}$

$= \dfrac{P(3\text{ yellow})}{P(\text{Same color})} = \dfrac{10/220}{15/220} = \dfrac{10}{15} = 2/3$

(c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

$P(\text{Orange} \geq 1) = 1 - P(\text{No orange})$

$= 1 - \left[\dfrac{8}{12} \times \dfrac{7}{11} \times \dfrac{6}{10}\right]$

$= 1 - \dfrac{336}{1320}$

$= \dfrac{41}{55}$

or

$P(\text{No orange}) = \dfrac{{}^{8}C_3}{{}^{12}C_3} = \dfrac{56}{220}$

$P(\text{At least one orange}) = 1 - \dfrac{56}{220}$

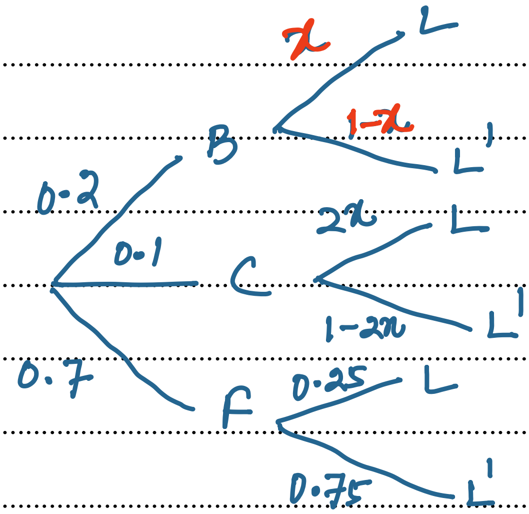
$= \dfrac{41}{55}$

6.

On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x. [3]



$P(L) = P(BL) + P(CL) + P(FL)$

$0.235 = 0.2x + 0.1 \times 2x + 0.7 \times 0.25$

$0.235 = 0.2x + 0.2x + 0.175$

$0.235 - 0.175 = 0.4x \Rightarrow x = \frac{0.06}{0.4} = \frac{3}{20} = 0.15$

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late. [2]

conditional probability

$P(C|L') = \frac{P(C \cap L')}{P(L')}$

$= \frac{0.1 \times (1 - 2 \times 0.15)}{1 - 0.235}$

$= \frac{0.07}{0.765} = \frac{14}{153}$

7.

(a) Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

No of arrangements $= \dfrac{9!}{2! \times 2!}$ 2T's 2A's

$= 90720$

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

5 letters between A's — No. of Arrangement

A _ _ _ _ _ A _ _ $7!/2!$

_ A _ _ _ _ _ A _ $7!/2!$

_ _ A _ _ _ _ _ A $7!/2!$

$\dfrac{7!}{2!} \times 3 = 7560$

6 letters between A's

A _ _ _ _ _ _ A _

_ A _ _ _ _ _ _ A

$\dfrac{7!}{2!} \times 2 = 5040$

7 letters Between A's

A _ _ _ _ _ _ _ A

$\dfrac{7!}{2!} \times 1 = 2520$

Total $= 7560 + 5040 + 250$

$= 15120$

Five letters are selected at random from the 9 letters in the word ACTIVATED.

(c) Find the probability that the selection does **not** contain more Ts than As. [5]

One T is choosen from Two Ts.

One A choosen from Two As

A	T	others
1	1	3
1	0	4
2	2	1
2	1	2
2	0	3
0	0	5

A T _ _ _ $\quad {}^5C_3 \times {}^2C_1 \times {}^2C_1 = 40$

A _ _ _ _ $\quad {}^5C_4 \times {}^2C_1 = 10$

A A T T _ $\quad {}^5C_1 = 5$

A A T _ _ $\quad {}^5C_2 \times {}^2C_1 = 20$

A A _ _ _ $\quad {}^5C_3 = 10$

_ _ _ _ _ $\quad {}^5C_5 = 1$

Total = 86

$$\text{Probability} = \frac{86}{{}^9C_5}$$

Total No. of Selections without Restrictions.

$$= \frac{86}{126}$$

$$= \frac{43}{63}$$

8.

A group of 15 friends visit an adventure park. The group consists of four families.

- Mr and Mrs Kenny and their four children
- Mr and Mrs Lizo and their three children
- Mrs Martin and her child
- Mr and Mrs Nantes

The group travel to the park in three cars, one containing 6 people, one containing 5 people and one containing 4 people. The cars are driven by Mr Lizo, Mrs Martin and Mr Nantes respectively.

(a) In how many different ways can the remaining 12 members of the group be divided between the three cars? [3]

6 Members — Mr. Lizo (Driver) (5 members)

5 Members — Mrs. Martin (Driver) (4 members)

4 members — Mr. Nantes (Driver) (3 members)

Number of ways : $^{12}C_5 \times {}^{7}C_4 \times {}^{3}C_3$

$= 792 \times 35 \times 1$

$= 27720$

The group enter the park by walking through a gate one at a time.

(b) In how many different orders can the 15 friends go through the gate if Mr Lizo goes first and each family stays together? [3]

Lizo's Family | Kenny's Family | Martin's | Nante's

L _ _ _ _ _ | _ _ _ _ _ _ | _ _ | _ _

Arrangement between them

Number of ways : $4! \times 6! \times 2! \times 2! \times 3!$

Lizo, Kenny, Martin, Nante

$24 \times 720 \times 2 \times 2 \times 6$

$= 414720$

In the park, the group enter a competition which requires a team of 4 adults and 3 children.

(c) In how many ways can the team be chosen from the group of 15 so that the 3 children are all from different families? [2]

Adults = 7   Children = 8

4 adults and 3 children are needed

(One from each family)

Number of ways:

1 from 4 children family

1 from 1 children family

$^7C_4 \times (^4C_1 \times {}^3C_1 \times {}^1C_1)$

1 from 3 children family

$35 \times 4 \times 3 \times 1$

$= 420$

(d) In how many ways can the team be chosen so that at least one of Mr Kenny or Mr Lizo is included? [3]

Total no. of ways (without restriction): $^7C_4 \times {}^8C_3 = 1960$

Two adults exclu

No. of ways (Neither Kenny nor Lizo is chosen): $^5C_4 \times {}^8C_3 = 280$

Required No. of ways (with at least one Mr. Kenny or Lizo): $1960 - 280$

$= 1680$

9.

A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

$$\text{Probability} = \frac{56}{500}$$

$$= \frac{14}{125}$$

conditional probability

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

and

$$P(\text{Devar} / \text{soccer}) = \frac{P(\text{Devar} \cap \text{Soccer})}{P(\text{Soccer})}$$

$$\frac{120/500}{280/500} = \frac{3}{7}$$

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

X = Students prefers Hockey

Y = Student is at Atmos College or Benn College

If X, Y are independent, then

$$P(X \cap Y) = P(X) \times P(Y)$$

Check:

$P(X \cap Y)$

$$P(X \text{ and } Y) = \frac{32+72}{500} = \frac{104}{500}$$

$$P(X) = \frac{220}{500}$$

$$P(Y) = \frac{86+156}{500} = \frac{242}{500}$$

$$P(X) \cdot P(Y) = P(X \cap Y)$$

$$\frac{220}{500} \times \frac{242}{500} \neq \frac{104}{500}$$

As $P(X) \cdot P(Y) \neq P(X \cap Y)$

X and Y are not Independent.

10.

The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

RESERVED   8 letters

2 R

3 E

S, V, D

V _ _ _ _ _ _ E

No of Arrangement : $\dfrac{6!}{2!\,2!}$

2R   2E (out of 3, 1E is fixed at the end)

No. of Arrangement without Restriction : $\dfrac{8!}{2! \times 3!}$

$$\text{Probability} = \frac{\frac{6!}{2! \times 2!}}{\frac{8!}{2! \times 3!}} = \frac{180}{3360} = \frac{3}{56}$$

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

$$P(2R \mid 3E) = \frac{P(2R \cap 3E)}{P(3E)}$$

conditional probability

P(2R and 3E)

$\boxed{R\ R}$ $\boxed{E\ E\ E}$ _ _ _

5!

P(3E)

$\boxed{E\ E\ E}$ _ _ R R _

$\dfrac{6!}{2!}$

$P(2R / 3E)$

$$= \frac{5!}{6!/2!}$$

$$= \frac{120}{360}$$

$$= \frac{1}{3}$$

11.

A summary of 40 values of x gives the following information:

$$\Sigma(x-k) = 520, \qquad \Sigma(x-k)^2 = 9640,$$

where k is a constant.

(a) Given that the mean of these 40 values of x is 34, find the value of k. [2]

$\bar{x} = 34$

$\Sigma(x-k) = 520$

$\Sigma x - nk = 520$ $\quad$ n = no. of Samples

$\Sigma x - 40k = 520$ $\quad$ mean of 40 values is 34

$1360 - 40k = 520$ $\quad$ $\frac{\Sigma x}{40} = 34$

$40k = 1360 - 520$

$40k = 840$ $\quad$ $\Sigma x = 34 \times 40$

$k = \frac{840}{40}$ $\quad$ $\Sigma x = 1360$

$k = 21$

(b) Find the variance of these 40 values of x. [2]

$$\text{Variance} = \frac{\Sigma(x-k)^2}{n} - \left(\frac{\Sigma(x-k)}{n}\right)^2$$

$$= \frac{9640}{40} - \left(\frac{520}{40}\right)^2$$

$$= 241 - 13^2$$

$$= 72$$

12.

(a) Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

Total No. of letters, $n = 8$

Repeated letters: 3 Os, 2 Rs

No. of arrangement $= \dfrac{8!}{3! \times 2!}$

$= 3360$

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

$R\ \underbrace{\_\ \_\ \_\ \_\ \_\ \_}\ R$

No. of arrangement $= \dfrac{6!}{3!} = 120$

for repeated 3Os

$R\ \underbrace{\_\ \boxed{O\ O\ O}\ \_\ \_}\ R$

No of arrangement (3 O's together)

$= 4! = 24$

Required no. of arrangement $= 120 - 24$

$= 96$

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

3 O's 2R's

O R _ _ $^3C_1 \times {}^2C_1 \times {}^3C_2 = 18$

O O R _ $^3C_2 \times {}^2C_1 \times {}^3C_1 = 18$

O O O R $^3C_3 \times {}^2C_1 \times {}^3C_0 = 2$

O R R _ $^3C_1 \times {}^2C_2 \times {}^3C_1 = 9$

O O R R $^3C_2 \times {}^2C_2 \times {}^3C_0 = 3$

Total possible Selections :

$18 + 18 + 2 + 9 + 3 = \underline{50}$