Answer ALL TWENTY FIVE questions.
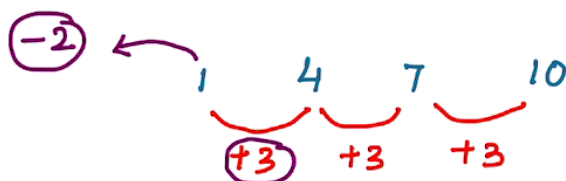
Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here are the first four terms of an arithmetic sequence.

$$1 \qquad 4 \qquad 7 \qquad 10$$

(a) Find an expression, in terms of n, for the nth term of this sequence.

-2

$1 \qquad 4 \qquad 7 \qquad 10$

$+3 \qquad +3 \qquad +3$

$n\text{th term} = 3n - 2$

(2)

The nth term of a different arithmetic sequence is $5n + 17$

(b) Find the 12th term of this sequence.

$n = 12$

$5\boxed{n} + 17$

$5 \times 12 + 17$

$60 + 17 = 77$

77

(1)

(Total for Question 1 is 3 marks)

2 450 students were asked how they travelled to school on Monday.
Each student walked or travelled by bus or travelled by car or travelled by bicycle.
Each student used just one method of travel.

One of these students is chosen at random.
The table shows information about the probability of each method of travel.

Method of travel	walk	bus	car	bicycle
Probability	0.20	x	$2x$	0.26

Work out how many of the 450 students travelled by car.

The total probability should be 1.

$0.20 + x + 2x + 0.26 = 1$ $\qquad$ P(car) $= 2x$

$\quad 3x + 0.46 = 1$ $\qquad$ P(car) $= 2 \times 0.18$

$\quad\quad 3x = 1 - 0.46$ $\qquad$ P(car) $= 0.36$

$\quad\quad 3x = 0.54$

$\quad\quad x = \dfrac{0.54}{3}$

$\quad\quad x = 0.18$

No of students travelled by Car

$\qquad 0.36 \times 450 = 162$

162

(Total for Question 2 is 4 marks)

3 Find the highest common factor (HCF) of 72 and 108
Show your working clearly.

72

6   12

2   3   3   4

2   2

108

2   54

2   27

3   9

3   3

2
2
2
3
3

2   3

→ H.C.F
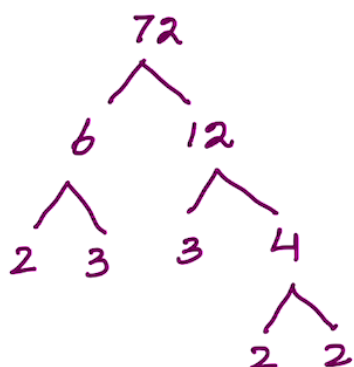
$72 = 2^3 \times 3^2$

$108 = 2^2 \times 3^3$

$H \cdot C \cdot F = 2^2 \times 3^2 = 36$
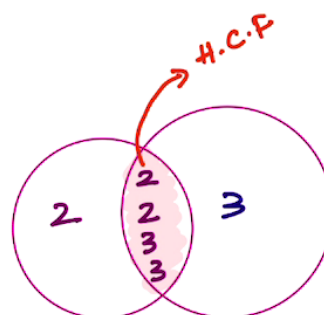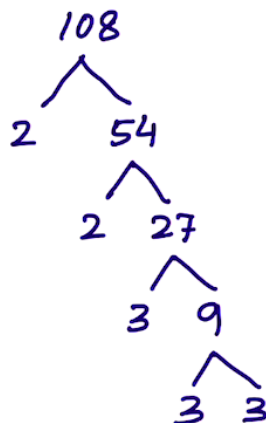
36

(Total for Question 3 is 2 marks)

4 Ava records the number of kilometres she drives each month.

In April, Ava drove 943 kilometres.
This is 15% more than the number of kilometres she drove in March.

Work out the number of kilometres Ava drove in March.

MARCH [?] → 100%

+ 15%

APRIL [943] → 115%

↳ multiplier $\frac{115}{100} = 1.15$

No of Kilometers drove in March

$943 \div 1.15 = 820 \text{ Km}$

820 Km .... kilometres

(Total for Question 4 is 3 marks)

5 In the diagram, *ABCDE* is a regular pentagon.

F

96°

→ This angle is to be found
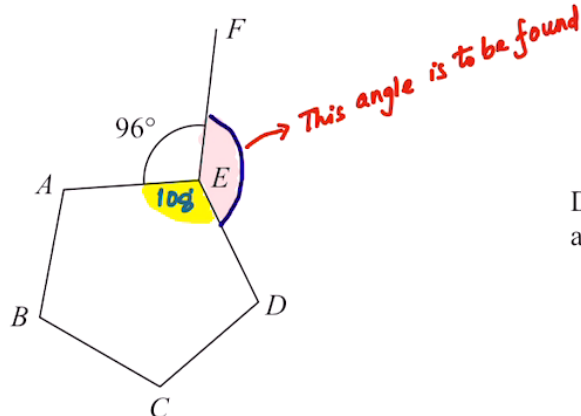
A

108

E

Diagram **NOT**
accurately drawn

B

D

C

Angle *AEF* = 96°

Work out the size of the obtuse angle *FED*
Show your working clearly.

Each interior angle of any regular polygon:

$$\frac{(n-2) \times 180°}{n}$$

For Pentagon
$n = 5$

$$\frac{(5-2) \times 180°}{5} = \underline{108°}$$

obtuse Angle FED + Angle AED + Angle AEF = 360°

obtuse Angle FED + 108° + 96° = 360°

obtuse Angle FED = 360° − 204°

= 156°

156° °

(Total for Question 5 is 4 marks)

6 (a) Expand and simplify $(m + 5)(m - 8)$

$$m^2 - 8m + 5m - 40$$
$$m^2 - 3m - 40$$

(b) Solve $3n - 4 = \dfrac{5n + 6}{3}$

Show clear algebraic working.

$$3n - 4 = \frac{5n+6}{3}$$
$$3(3n-4) = 5n+6$$
$$9n - 12 = 5n+6$$
$$9n - 5n = 6+12$$
$$4n = 18$$
$$n = \frac{18}{4} = \frac{9}{2}$$

$n = \dfrac{9}{2}$

(Total for Question 6 is 5 marks)

7 $\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$

$A = \{\text{even numbers}\}$

$B = \{23, 29, 31\}$

$C = \{\text{multiples of 3}\}$

First list the numbers

$C = \{24, 27, 30, 33\}$

$A = \{24, 26, 28, 30, 32, 34\}$

(a) List the members of the set

  (i) $B \cup C$ *List all the numbers of SET B & SET C*

$$\{23, 24, 27, 29, 30, 31, 33\}$$

(1)

$A = \{\text{Even numbers}\}$

$A' = \{\text{Not Even Numbers or ODD Numbers}\}$

  (ii) $A' \cap C$

$A' = \{23, 25, 27, 29, 31, 33\}$

$C = \{24, 27, 30, 33\}$

$$\{27, 33\}$$

(1)

(b) Is it true that $B \cap C = \varnothing$?

Tick (✓) one of the boxes below.

Yes ✓   No ☐

Give a reason for your answer.

There are no multiples of 3 in SET B.

(1)

The set D has 4 members and is such that $D \cap (A \cup C) = \varnothing$

(c) List the members of set D *There shouldn't be any Common element between D and A∪C*

$A \cup C = \{24, 26, 27, 28, 30, 32, 33, 34\}$

$$D = \{23, 25, 29, 31\}$$

(2)

(Total for Question 7 is 5 marks)

8 A cylinder is placed on a table.

21 cm   →h
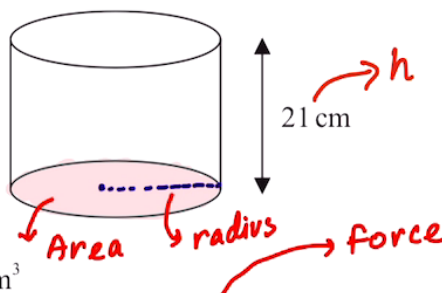
→Area   →radius   →force

Diagram **NOT** accurately drawn

The volume of the cylinder is 1575 cm^3
The force exerted by the cylinder on the table is 84 newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the table due to the cylinder.

Volume of Cylinder $= 1575$

$\pi r^2 h = 1575$

$\pi \times r^2 \times 21 = 1575$

$r^2 = \dfrac{1575}{\pi \times 21}$

$r = \sqrt{\dfrac{1575}{\pi \times 21}}$

radius $(r) = 4.886$

Area of base of Cylinder:

$\pi r^2 = \pi \times (4.886)^2$

$\text{Pressure} = \dfrac{\text{Force}}{\text{Area}}$

$\text{Pressure} = \dfrac{84}{\pi \times (4.886)^2}$

$= 1.129$

.... **1.129** .... newtons/cm^2

(Total for Question 8 is 3 marks)

9 The table gives the amount of rice produced by each of two countries in 2020

Country	Amount of rice (tonnes)
Indonesia	3.5×10^7
Argentina	8.2×10^5

(a) Write 3.5×10^7 as an ordinary number.

35 000 000

(1)

In 2020, Japan produced 6 780 000 more tonnes of rice than Argentina.

(b) Work out the amount of rice Japan produced in 2020
Give your answer in standard form.

Rice produced by Japan :

$8.2 \times 10^5 + 6780000$

$= 820000 + 6780000$

$= 7600000$

$= 7.6 \times 10^6$

7.6×10^6 tonnes

(2)

(Total for Question 9 is 3 marks)

10 (a) Simplify $(2p)^0$ where $p > 0$

Rule:
$$x^0 = 1$$

.... 1 ....

(1)

$y^9 \times y^{-3} = y^n$

(b) Find the value of n

$$y^9 \times y^{-3} = y^n$$
$$y^{9+(-3)} = y^n$$
$$y^6 = y^n \qquad \boxed{n = 6}$$

Rule: $\boxed{x^m \times x^n = x^{m+n}}$

If $x^m = x^n$
then $m = n$

$n = $ .... 6 ....

(1)

(c) Simplify fully $(5a^4c^2)^3$

$$5^3 \times (a^4)^3 (c^2)^3$$
$$125 \, a^{12} c^6$$

Rule: $(xy)^m = x^m \times y^m$

$(x^m)^n = x^{mn}$

.... $125 a^{12} c^6$ ....

(2)

(Total for Question 10 is 4 marks)

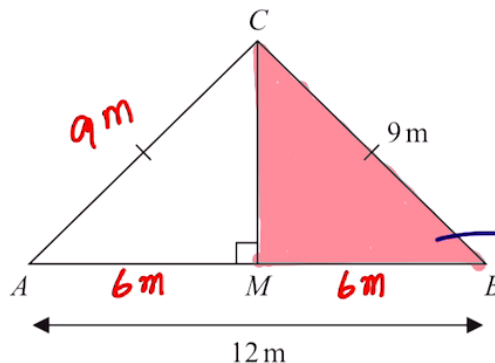**11** The diagram shows a roof support.

Diagram **NOT** accurately drawn

The roof support is made from four lengths of wood, AB, AC, BC and MC

$AC = BC = 9\,\text{m}$ $AB = 12\,\text{m}$

angle $AMC = 90°$

Lewis is going to buy lengths of wood to make the roof support.

The wood costs 21.50 euros per metre.
Each length of wood he buys has to be a whole number of metres.

Work out the total cost of the wood Lewis needs to buy.
Show your working clearly.

CMB is a right angled triangle. Using Pythagora's theorem

$$CM^2 + MB^2 = CB^2$$
$$CM^2 + 6^2 = 9^2$$
$$CM = \sqrt{9^2 - 6^2}$$
$$CM = 6.7 \simeq 7\,m \quad \rightarrow \text{nearest whole number}$$

Total wood needed: $9 + 9 + 12 + 7 = \underline{37\,m}$

Total Cost: $37 \times 21.50 = 795.50$ euros

_____795.50_____ euros

(Total for Question 11 is 4 marks)

12 (a) Factorise fully $\quad 6y^2 - 5y - 4$

$$\left(\overset{3}{6y} - \overset{4}{8}\right)\left(\overset{2}{6y} + \overset{1}{3}\right)$$

$a = 6$
$b = -5 \checkmark$
$c = -4$

$$\frac{-8}{-8} \times \frac{3}{3} = -24$$
$$-8 + 3 = -5$$

$$(3y - 4)(2y + 1)$$

Alternatively:

$$6y^2 - 5y - 4$$
$$6y^2 - 8y + 3y - 4$$
$$2y(3y - 4) + 1(3y - 4)$$

$(3y-4)(2y+1)$
(2)

$$= (3y - 4)(2y - 1)$$

(b) Express $\dfrac{2x + 1}{4x} + \dfrac{7 - 5x}{3x}$ as a single fraction in its simplest form.

$$\frac{3x(2x+1) + 4x(7-5x)}{4x \times 3x}$$

$$\frac{6x^2 + 3x + 28x - 20x^2}{12x^2}$$

Rule:
$$\frac{a}{b} + \frac{c}{d}$$
$$\frac{ad + bc}{bd}$$

$$\frac{31x - 14x^2}{12x^2}$$

$$\frac{x(31 - 14x)}{12 \times x \times x}$$

$$\frac{31 - 14x}{12x}$$

$$\frac{31 - 14x}{12x}$$
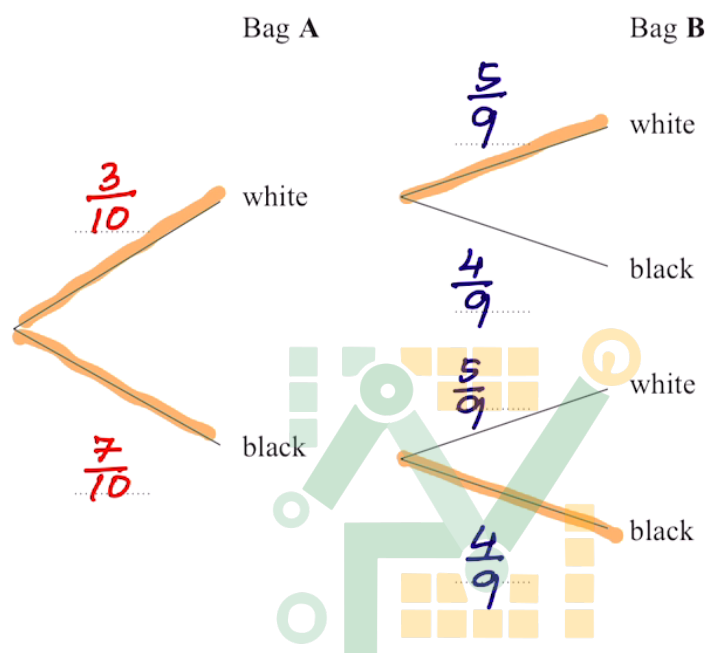(3)

(Total for Question 12 is 5 marks)

13 Harman has two bags of beads.

In bag **A**, there are 3 white beads and 7 black beads. *Total 10 Beads*
In bag **B**, there are 5 white beads and 4 black beads. *Total 9 Beads*

Harman takes at random a bead from bag **A** and a bead from bag **B**

(a) Complete the probability tree diagram.

Bag **A**   Bag **B**

$\frac{3}{10}$ white

$\frac{5}{9}$ white

$\frac{4}{9}$ black

$\frac{7}{10}$ black

$\frac{5}{9}$ white

$\frac{4}{9}$ black

(2)

(b) Work out the probability that Harman takes two beads of the same colour.

$$P(White, White) + P(Black, Black)$$

$$\left(\frac{3}{10} \times \frac{5}{9}\right) + \left(\frac{7}{10} \times \frac{4}{9}\right)$$

$$\frac{1}{6} + \frac{14}{15}$$

$$\frac{43}{90}$$

$$\frac{43}{90}$$

(3)

(Total for Question 13 is 5 marks)

14 The combined savings of Abel and Bahira are 15 435 dinars.

The savings of Bahira are 45% more than the savings of Abel.

The savings of Bahira are $\dfrac{3}{2}$ times the savings of Chanda.

Work out the savings of Chanda.

let Savings of Abel $= x$

Then Savings of Bahira $= x + $ 45% of x

$\qquad\qquad\qquad = x + 0.45x$

$\qquad\qquad\qquad = 1.45x$

Savings of Abel + Savings of Bahira $= 15435$

$\qquad x + 1.45x = 15435$

$\qquad\qquad 2.45x = 15435$

$\qquad\qquad x = \dfrac{15435}{2.45}$

$\qquad\qquad x = 6300$ dinar.

Savings of Bahira $= 1.45x = 1.45 \times 6300 = 9135$

Savings of Bahira $= \dfrac{3}{2} \times$ Savings of chanda

$\qquad 9135 = 1.5 \times$ Savings of chanda

Savings of chanda $= \dfrac{9135}{1.5} = 6090$ dinar

......6090...... dinars

(Total for Question 14 is 5 marks)

15 The function f is defined as

$$f : x \mapsto \frac{3x + 1}{x - 2}$$

(a) State the value of x that cannot be included in any domain of the function f

$$x = 2$$

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$$y = \frac{3x+1}{x-2}$$

$$y(x-2) = 3x+1$$

$$xy - 2y = 3x+1$$

$$xy - 3x = 1 + 2y$$

$$x(y-3) = 1 + 2y$$

$$x = \frac{1+2y}{y-3}$$

$$f^{-1}(x) = \frac{1+2x}{x-3}$$

$$f^{-1}(x) = \frac{1+2x}{x-3}$$

(3)

(Total for Question 15 is 4 marks)

16 There are 20 sweets in a box.

15 of the sweets are red
5 of the sweets are yellow

Fred takes at random 3 sweets from the box.

Work out the probability that Fred takes at least one sweet of each colour from the box.

$$P(Red) = \frac{15}{20} \qquad P(Yellow) = \frac{5}{20}$$

Probability of getting Same colour:

$$P(Red, Red, Red) + P(Yellow, Yellow, Yellow)$$

$$\left(\frac{15}{20} \times \frac{14}{19} \times \frac{13}{18}\right) + \left(\frac{5}{20} \times \frac{4}{19} \times \frac{3}{18}\right)$$

$$\frac{91}{228} + \frac{1}{114}$$

$$\frac{31}{76}$$

Probability of getting at least one sweet of each colour:

$$1 - \text{Probability of getting Same colour}$$

$$1 - \frac{31}{76}$$

$$\frac{45}{76}$$

$$\frac{45}{76}$$

(Total for Question 16 is 4 marks)

17 Show that $\dfrac{1 + \sqrt{5}}{3 - \sqrt{5}}$ can be written in the form $a + \sqrt{b}$ where a and b are integers.

Show each stage of your working clearly.

$$\frac{(1+\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

Multiply by the conjugate of denominator

$$(a+b)(a-b) = a^2 - b^2$$

$$\frac{3+\sqrt{5} + 3\sqrt{5} + 5}{3^2 - (\sqrt{5})^2}$$

$$\frac{8 + 4\sqrt{5}}{9 - 5}$$

$$\frac{8 + 4\sqrt{5}}{4} = \frac{4(2+\sqrt{5})}{4} = 2+\sqrt{5}$$

(Total for Question 17 is 3 marks)

18 A curve C has equation $y = x^3 - 40x + 1$

Find the coordinates of both the points on C at which the gradient is 8

$$\frac{dy}{dx} = 8$$

$y = x^3 - 40x + 1$

$\frac{dy}{dx} = 3x^2 - 40$

$3x^2 - 40 = 8$

$3x^2 = 40 + 8$

$3x^2 = 48$

$x^2 = \frac{48}{3} = 16$

$x = \sqrt{16} = \pm 4$

$x = 4$	$x = -4$
$y = x^3 - 40x + 1$	$y = x^3 - 40x + 1$
$y = 4^3 - (40 \times 4) + 1$	$y = (-4)^3 - 40(-4) + 1$
$y = -95$	$y = 97$

(4 , −95)

(−4 , 97)

(Total for Question 18 is 5 marks)

19 Here is quadrilateral *ABCD*
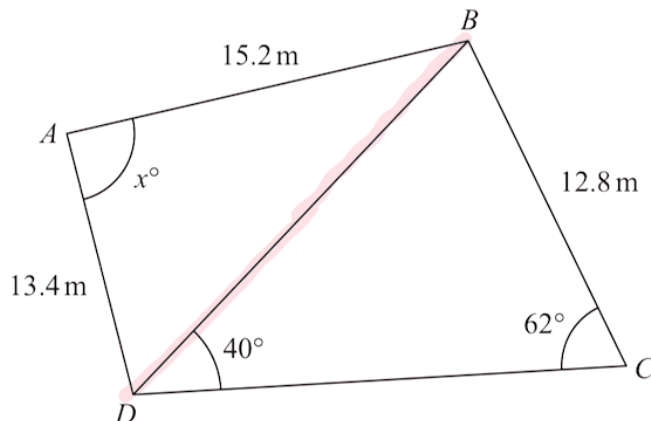
B
15.2 m
A
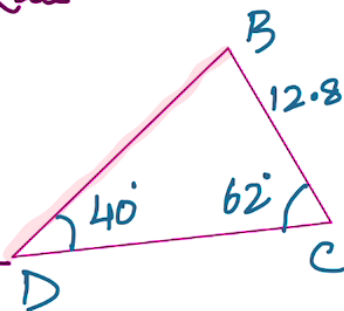x°
12.8 m
Diagram **NOT**
accurately drawn
13.4 m
62°
40°
C
D

Work out the value of *x*
Give your answer correct to 3 significant figures.
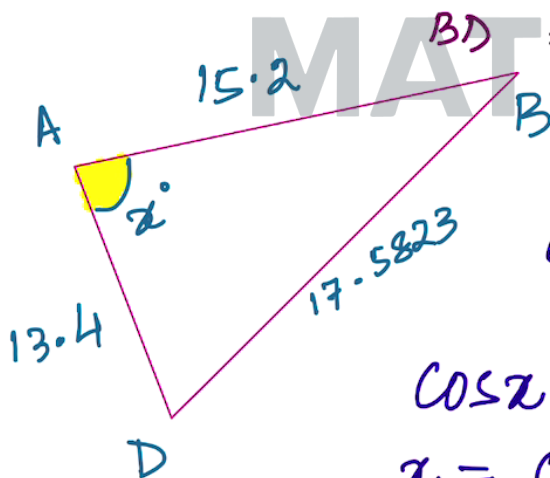
To find length of BD use Sine Rule.

$$\frac{BD}{\sin 62} = \frac{12\cdot8}{\sin 40}$$

$$BD = \frac{12\cdot8 \times \sin 62}{\sin 40}$$

B
12.8
40°
62°
D
C

$$BD = 17\cdot5823 \ m$$

15.2
A
B
x°
17.5823
13.4
D

To find x use Cosine Rule

$$\cos x = \frac{15\cdot2^2 + 13\cdot4^2 - 17\cdot5823^2}{2 \times 15\cdot2 \times 13\cdot4}$$

$$\cos x = 0\cdot24907$$

$$x = \cos^{-1}(0\cdot24907)$$

x = ___75·6___

$$x = 75\cdot577$$

(Total for Question 19 is 5 marks)

$$x = 75\cdot6 \ (3 \ sig.)$$

20 The diagram shows a sector $OABC$ of a circle centre O

A

r

O 60°

r

B

C

Diagram **NOT** accurately drawn
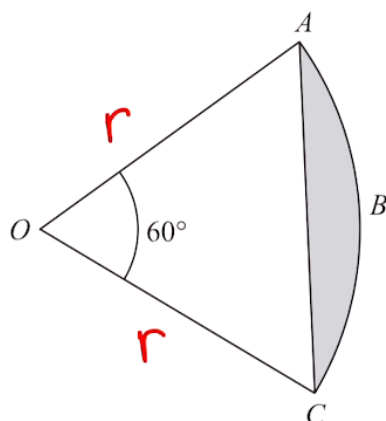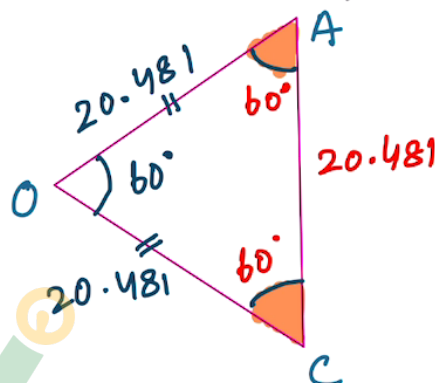
It is an isosceles triangle, two angles are equal, but other two angles are each 60°
→ So it becomes Equilateral triangle
So $AC = 20.481$

20.481

A
60°

O 60°

20.481

60°

20.481

C

Angle $AOC = 60°$

The area of the shaded segment ABC is $38\,\text{cm}^2$

Work out the perimeter of the shaded segment ABC
Give your answer correct to one decimal place.

Area of Sector $OABC$ — Area of triangle OAC = Area of Shaded

$$\frac{60°}{360} \times \pi r^2 \quad - \quad \frac{1}{2} \times r^2 \times \sin 60° \quad = \quad 38\ \text{cm}^2$$

$$\frac{1}{6} \times \pi r^2 \quad - \quad \frac{1}{2} \times r^2 \times \frac{\sqrt{3}}{2} \quad = 38$$

$$r^2 \left[\frac{1}{6}\pi - \frac{1}{2} \times \frac{\sqrt{3}}{2} \right] = 38$$

$$r^2 \times 0.090586 = 38$$

$$r^2 = \frac{38}{0.090586}$$

$$r = \sqrt{\frac{38}{0.090586}}$$

$$r = 20.481$$

Arc length ABC

$$\frac{60}{360} \times 2\pi \times 20.481$$

$$= 21.44765$$

Perimeter of Shaded

Arc length ABC + length AC

$$21.44765 + 20.481$$

$$41.9\ (1\,dp)$$

_____ 41.9 _____ cm

(Total for Question 20 is 4 marks)

21 A curve has equation $y = f(x)$

There is one minimum point on this curve.
The coordinates of this minimum point are $(5, -4)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 7)$

Rule:
$y = f(x)$ (x, y)
$y = f(x+a)$ $(x-a, y)$

$(5, -4)$
$(5-7, -4)$
$(-2, -4)$

(__-2__ , __-4__)

(1)

(ii) $y = f(x) - 6$

Rule:
$y = f(x)$ (x, y)
$y = f(x) \pm b$ $(x, y \pm b)$

$(5, -4)$
$(5, -4-6)$
$(5, -10)$

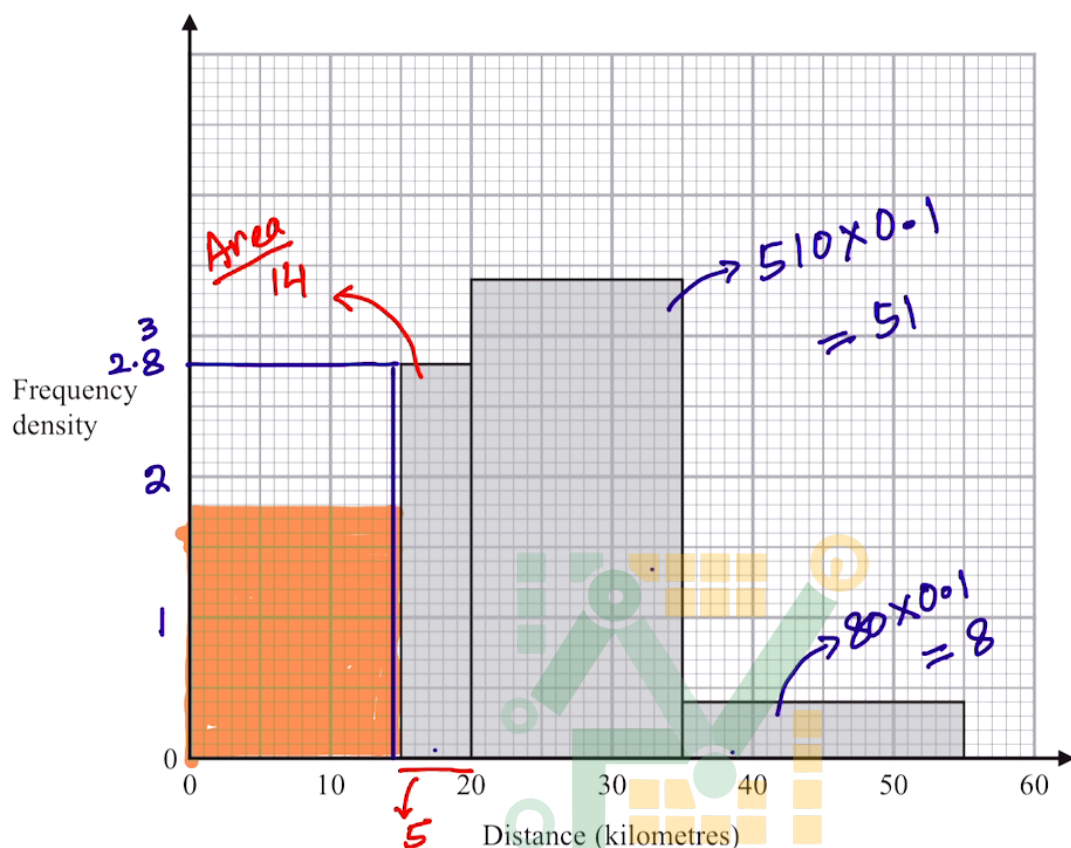(__5__ , __-10__)

(1)

(Total for Question 21 is 2 marks)

22 The incomplete histogram shows some information about the distances, in kilometres, that 100 adults ran last week.

Area
14

510×0.1
$= 51$

Frequency density

80×0.1
$= 8$

5

Distance (kilometres)

All of the adults ran at least 5 kilometres.
None of the adults ran more than 55 kilometres.
14 adults ran between 15 kilometres and 20 kilometres.

Complete the histogram.

Height of bar of $(15-20) = \dfrac{14}{5} = 2.8$

140 Small squares $= 14$

1 Small Square $= \dfrac{14}{140} = 0.1$

10 Small Square $= 1$ adult

No of adults shown in the diagram
$14 + 51 + 8 = 73$

Remaining to show: $100 - 73 = 27$ (270 small squares)

(Total for Question 22 is 3 marks)

23 A solid shape is made by removing a hemisphere, shown shaded, from a cone as shown in the diagram.
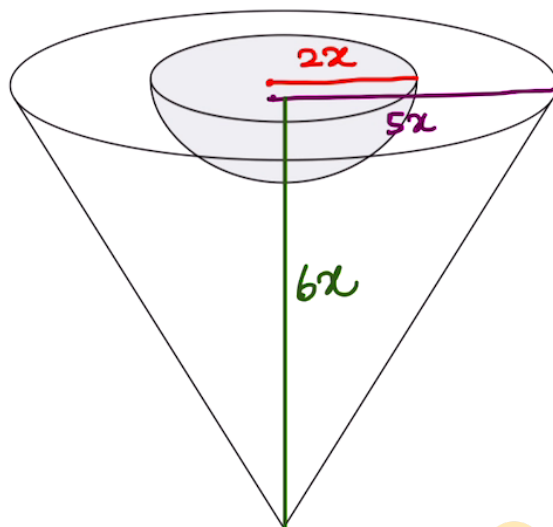
2x

5x

6x

Diagram **NOT** accurately drawn

The radius of the hemisphere is $2x$ cm
The radius of the base of the cone is $5x$ cm
The vertical height of the cone is $6x$ cm

The volume of the solid shape is 6948π cm^3

Work out the **total** surface area of the solid hemisphere that has been removed from the cone.
Give your answer correct to the nearest integer.

volume of Cone — volume of half sphere = volume of Solid

$$\frac{1}{3}\pi \times (5x)^2 \times (6x) - \frac{1}{2} \times \frac{4}{3} \times \pi \times (2x)^3 = 6948\pi$$

$$\frac{1}{3} \times \pi \times 25x^2 \times 6x - \frac{1}{2} \times \frac{4}{3} \times \pi \times 8x^3 = 6948\pi$$

$$50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$$

$$\pi x^3 \left[50 - \frac{16}{3}\right] = 6948\pi$$

$$x^3 \times \frac{134}{3} = 6948$$

$$x^3 = \frac{6948 \times 3}{134}$$

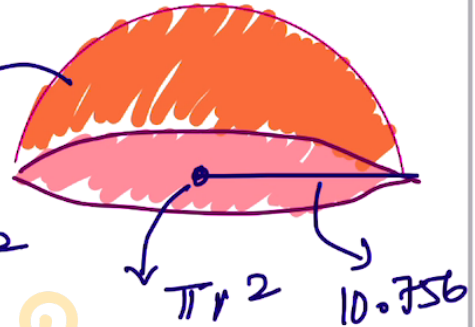$$x = \sqrt[3]{\frac{6948 \times 3}{134}}$$

$$x = 5.378$$

Radius of hemisphere

$$2x = 2 \times 5.378 = 10.756 \, cm$$

Total Surface Area
of Hemisphere $2\pi r^2$

$$2\pi r^2 + \pi r^2 = 3\pi r^2$$

πr^2 10.756

$$3 \times \pi \times (10.756)^2 = 1090.36$$

$$\underline{1090} \quad \text{cm}^2$$

(Total for Question 23 is 5 marks)

24 A polygon has n sides, where $n > 5$

The interior angles of the polygon form an arithmetic sequence.

The smallest angle of the polygon is 84° $\longrightarrow a = 84°$
The common difference of the sequence is 4° $\rightarrow d = 4°$

Work out the sum of the interior angles of the polygon.
Show clear algebraic working.

$a = 84°$ $d = 4°$

S_n formula in Arithmetic Series

formula to find sum of angles of any polygon

$$\frac{n}{2}\left[2a + (n-1)d\right] = (n-2) \times 180$$

$$\frac{n}{2}\left[2 \times 84 + (n-1)4\right] = (n-2) \times 180$$

$$\frac{n}{2}\left[168 + 4n - 4\right] = 180n - 360$$

$$n(168 + 4n - 4) = 2(180n - 360)$$

$$168n + 4n^2 - 4n = 360n - 720$$

$$4n^2 + 168n - 4n - 360n + 720 = 0$$

$$4n^2 - 196n + 720 = 0$$

(Divide each term by 4)

$$n^2 - 49n + 180 = 0$$

$a = 1, \ b = -49 \ \ c = 180$

$$n = \frac{-(-49) \pm \sqrt{(-49)^2 - 4 \times 1 \times 180}}{2}$$

$n = \ 45$ or $n = 4$

$\rightarrow$ Ignore this value because it the question it's given $n > 5$

Sum of all interior angles

$(n-2) \times 180$

$(45-2) \times 180$
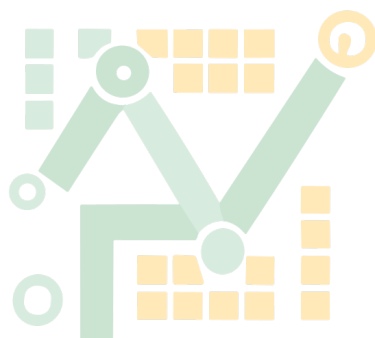
7740

7740

(Total for Question 24 is 6 marks)

25 $f(x) = 17 - 3x^2 + 12x$

Write $f(x)$ in the form $a - b(x - c)^2$ where a, b and c are constants.

$$f(x) = 17 - 3x^2 + 12x$$
$$= -3x^2 + 12x + 17$$
$$= -3[x^2 - 4x] + 17$$
$$= -3[(x-2)^2 - 2^2] + 17$$
$$= -3(x-2)^2 + 12 + 17$$
$$= -3(x-2)^2 + 29$$
$$= 29 - 3(x-2)^2$$

Completing Square

$$x^2 \pm px + q$$
$$\left(x \pm \frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q$$

$f(x) = \underline{29 - 3(x-2)^2}$

(Total for Question 25 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS