Cambridge International AS & A Level

CANDIDATE NAME SOLVED BY MR. PABITRA

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

1 In the expansion of $\left(kx+\frac{2}{x}\right)^4$, where k is a positive constant, the term independent of x is equal to 150.

Find the value of k and hence determine the coefficient of x^2 in the expansion. [4]

Binomial Expansion formula:

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots$$

$$\left(kx+\frac{2}{x}\right)^4 = (kx)^4 + \underbrace{{}^4C_1 (kx)^3 \left(\frac{2}{x}\right)} + \underbrace{{}^4C_2 (kx)^2 \left(\frac{2}{x}\right)^2} + \dots$$

Independent of x

$${}^4C_2\, k^2 \cancel{x^2} \times \frac{4}{\cancel{x^2}}$$

$$6k^2 \times 4$$

$$24k^2$$

$$24k^2 = 150$$

$$k^2 = \frac{150}{24}$$

$$k = \sqrt{\frac{150}{24}}$$

$$k = \frac{5}{2}$$

(Given that k is a positive constant, so negative value is ignored)

Term in x^2:

$${}^4C_1 (kx)^3 \left(\frac{2}{x}\right)$$

$$4 \times k^3 \times x^3 \times \frac{2}{x}$$

$$8k^3 x^2$$

Coefficient of x^2

$$8k^3$$

$$= 8 \times \left(\frac{5}{2}\right)^3$$

$$= 125$$

2 The curve $y = x^2 - \frac{a}{x}$ has a stationary point at $(-3, b)$.

Find the values of the constants a and b. [4]

At stationary point, the gradient of the Curve $= 0$

$$\frac{dy}{dx} = 0$$

$$y = x^2 - \frac{a}{x}$$

$$y = x^2 - ax^{-1}$$

$$\frac{dy}{dx} = 2x - (-1)ax^{-2}$$

$$\frac{dy}{dx} = 2x + \frac{a}{x^2}$$

$$2x + \frac{a}{x^2} = 0$$

Multiplying x^2 with each term

$$2x^3 + a = 0$$

at $(-3,b)$: $2(-3)^3 + a = 0$

$$-54 + a = 0$$

$$a = 54$$

To find the value of b:

$$y = x^2 - \frac{a}{x}$$

at $(-3,b)$

$$b = (-3)^2 - \frac{54}{(-3)}$$

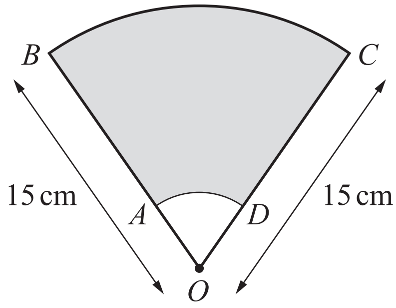
$$b = 9 + \frac{54}{3}$$

$$b = 9 + 18$$

$$b = 27$$

$a = 54 \quad b = 27$

3



The diagram shows a sector of a circle, centre O, where $OB = OC = 15$ cm. The size of angle BOC is $\frac{2}{5}\pi$ radians. Points A and D on the lines OB and OC respectively are joined by an arc AD of a circle with centre O. The shaded region is bounded by the arcs AD and BC and by the straight lines AB and DC. It is given that the area of the shaded region is $\frac{209}{5}\pi\ \text{cm}^2$.

Find the perimeter of the shaded region. Give your answer in terms of π. [5]

Shaded Area = Area of Sector OBC − Area of Sector OAD

$$\frac{209}{5}\pi = \frac{1}{\cancel{2}} \times 15^2 \times \frac{\cancel{2}\pi}{5} - \frac{1}{\cancel{2}} \times r^2 \times \frac{\cancel{2}\pi}{5}$$

$$\frac{209}{5}\pi = \frac{225\pi}{5} - \frac{r^2\pi}{5}$$

$$\cancel{\frac{\pi}{5}} \times 209 = \cancel{\frac{\pi}{5}}\left[225 - r^2\right]$$

$$209 = 225 - r^2$$

$$r^2 = 225 - 209$$

$$r = \sqrt{16}$$

$r = 4$

$OA = OD = 4$ cm

$AB = CD = 15 - 4 = 11$ cm

$\boxed{\text{Formula: Arc length} = r\theta \quad \text{Area of Sector} = \frac{1}{2}r^2\theta}$

Perimeter of shaded Region:

$\overline{AB} + \text{Arc } BC + \overline{CD} + \text{arc } AD$

$$11 + \left(15 \times \frac{2\pi}{5}\right) + 11 + \left(4 \times \frac{2\pi}{5}\right)$$

$$11 + \frac{30\pi}{5} + 11 + \frac{8\pi}{5}$$

$$\left(22 + \frac{38}{5}\pi\right) \text{ cm}$$

4 Show that the curve with equation $x^2 - 3xy - 40 = 0$ and the line with equation $3x + y + k = 0$ meet for all values of the constant k. [5]

For intersection, Solve as Simultaneous Equations

$x^2 - 3xy - 40 = 0$ $3x + y + k = 0$

$x^2 - 3x(-3x - k) - 40 = 0$ $y = -3x - k$

$x^2 + 9x^2 + 3kx - 40 = 0$

$\underbrace{10}_{a}x^2 + \underbrace{3k}_{b}x \underbrace{- 40}_{c} = 0$

$b^2 - 4ac = (3k)^2 - 4(10)(-40)$

$b^2 - 4ac = 9k^2 + 1600$

For all values of k, $9k^2 \geq 0$ because k^2 is a square term.

So, $b^2 - 4ac > 0$ for all values of k.

The line and curve meet for all values of constant k.

5 The equation of a curve is such that $\frac{dy}{dx} = 4x - 3\sqrt{x} + 1$.

(a) Find the x-coordinate of the point on the curve at which the gradient is $\frac{11}{2}$. [3]

$\frac{dy}{dx} = 4x - 3\sqrt{x} + 1$

$\frac{11}{2} = 4x - 3\sqrt{x} + 1$

$4x - 3\sqrt{x} + 1 - \frac{11}{2} = 0$

$4x - 3\sqrt{x} - \frac{9}{2} = 0$

Multiply by 2 with each term

$8x - 6\sqrt{x} - 9 = 0$

let $y = \sqrt{x}$, $y^2 = x$

$8y^2 - 6y - 9 = 0$

$(4y+3)(2y-3) = 0$

$4y + 3 = 0$

$y = -3/4$

$\sqrt{x} = -3/4$

$x = (-3/4)^2 = 9/16$

$2y - 3 = 0$

$y = 3/2$

$\sqrt{x} = 3/2$

$x = (3/2)^2 = 9/4$

Checking x-values

when $x = 9/16$

$\frac{dy}{dx} = 4\left(\frac{9}{16}\right) - 3\sqrt{9/16} + 1$

$\frac{dy}{dx} = 1$

when $x = 9/4$

$\frac{dy}{dx} = 4(9/4) - 3\sqrt{9/4} + 1$

$\frac{dy}{dx} = 11/2$ ✓

So $x = \frac{9}{4}$

(b) Given that the curve passes through the point (4, 11), find the equation of the curve. [4]

$\frac{dy}{dx} = 4x - 3\sqrt{x} + 1$

$\int dy = \int (4x - 3\sqrt{x} + 1)\, dx$

$3x^{1/2}$

$y = \frac{4x^2}{2} - \frac{3x^{3/2}}{3/2} + x + c = 2x^2 - 2x^{3/2} + x + c$

at $(4, 11)$ $\quad 11 = 2(4)^2 - 2(4)^{3/2} + 4 + c$

$11 = 32 - 16 + 4 + c$

$11 = 20 + c$

$c = -9$

So, Equation of the Curve is : $y = 2x^2 - 2x^{3/2} + x - 9$

6 Circles C_1 and C_2 have equations

$$x^2+y^2+6x-10y+18=0 \quad \text{and} \quad (x-9)^2+(y+4)^2-64=0$$

respectively.

(a) Find the distance between the centres of the circles. [4]

Equation of a Circle
$(x-a)^2+(y-b)^2=r^2$
Centre (a,b)

$x^2+y^2+6x-10y+18=0$

$x^2+6x+y^2-10y+18=0$

$(x+3)^2-3^2+(y-5)^2-5^2+18=0$

$(x+3)^2+(y-5)^2-9-25+18=0$

$(x+3)^2+(y-5)^2-16=0$

$(x+3)^2+(y-5)^2=16$

Centre of C_1 $(-3,5)$ Radius $=4$

$(x-9)^2+(y+4)^2-64=0$

$(x-9)^2+(y+4)^2=64$

Centre of C_2 $(9,-4)$ Radius $=8$

Distance between Centres: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(9-(-3))^2+(-4-5)^2}=\sqrt{12^2+(-9)^2}=\sqrt{225}$

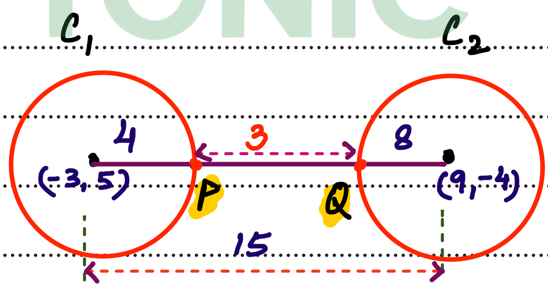
$=15$

P and Q are points on C_1 and C_2 respectively. The distance between P and Q is denoted by d.

(b) Find the greatest and least possible values of d. [3]

Radius of $C_1=4$ Radius of $C_2=8$

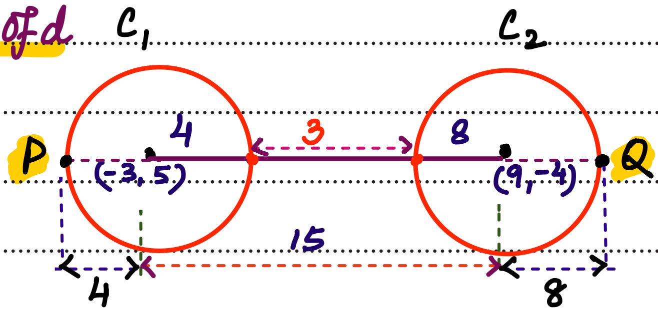
Least possible value of d



$d=15-(4+8)$

$d=3$

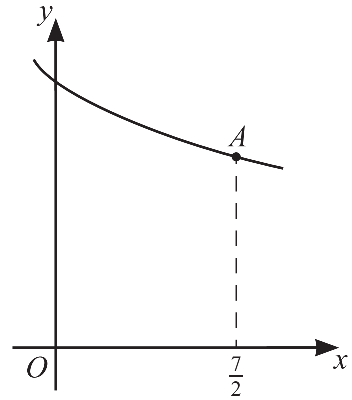
Greatest possible value of d



$d=4+15+8$

$d=27$

7



The diagram shows part of the curve with equation $y = \dfrac{12}{\sqrt[3]{2x+1}}$. The point A on the curve has coordinates $\left(\frac{7}{2}, 6\right)$.

(a) Find the equation of the tangent to the curve at A. Give your answer in the form $y = mx + c$. [4]

$$y = \frac{12}{\sqrt[3]{2x+1}}$$

Rule: $\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1} \times a$

$$y = 12(2x+1)^{-1/3}$$

$$\frac{dy}{dx} = 12 \times \frac{-1}{3}(2x+1)^{-1/3-1} \times 2$$

$$\frac{dy}{dx} = -8(2x+1)^{-4/3}$$

At $\left(\frac{7}{2}, 6\right)$ $\quad \frac{dy}{dx} = -8\left(2 \times \frac{7}{2} + 1\right)^{-4/3}$

$$\frac{dy}{dx} = \frac{-8}{8^{4/3}} = \frac{-8}{16}$$

Gradient of tangent $\quad \frac{dy}{dx} = -\frac{1}{2}$

Equation of tangent:

$$y - y_1 = m(x - x_1)$$

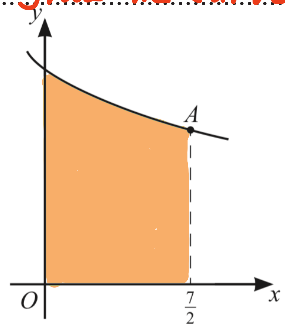
$$y - 6 = -\frac{1}{2}\left(x - \frac{7}{2}\right)$$

$$y - 6 = -\frac{x}{2} + \frac{7}{4}$$

$$y = \frac{-x}{2} + \frac{7}{4} + 6$$

$$y = \frac{-1}{2}x + \frac{31}{4}$$

(b) Find the area of the region bounded by the curve and the lines $x = 0$, $x = \frac{7}{2}$ and $y = 0$. [4]



Area of Region : (To find the Area integrate the Curve)

$$\int_0^{7/2} \frac{12}{\sqrt[3]{2x+1}} \cdot dx$$

$$\int_0^{7/2} 12(2x+1)^{-1/3} \cdot dx$$

$$12\left[\frac{(2x+1)^{-\frac{1}{3}+1}}{\left(-\frac{1}{3}+1\right)\times 2}\right]_0^{7/2}$$

$$12\left[\frac{(2x+1)^{2/3}}{4/3}\right]_0^{7/2}$$

$$12 \times \frac{3}{4}\left[(2x+1)^{2/3}\right]_0^{7/2}$$

$$9\left[\left(2\times\tfrac{7}{2}+1\right)^{2/3} - (2\times 0+1)^{2/3}\right]$$

$$9\left[8^{2/3} - 1^{2/3}\right]$$

$$9[4-1]$$

$$9 \times 3 = 27$$

Rule :

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}$$

8 (a) It is given that β is an angle between 90° and 180° such that $\sin\beta = a$.

Express $\tan^2\beta - 3\sin\beta\cos\beta$ in terms of a. [3]

$\tan^2\beta - 3\sin\beta\cos\beta$

$\frac{\sin^2\beta}{\cos^2\beta} - 3\sin\beta\cos\beta$

$\frac{a^2}{1-a^2} - 3a\sqrt{1-a^2}$

$\sin^2\beta + \cos^2\beta = 1$

$\cos^2\beta = 1 - \sin^2\beta$

$\cos^2\beta = 1 - a^2$

$\cos\beta = \sqrt{1-a^2}$

Answer:

$\frac{a^2}{1-a^2} - 3a\sqrt{1-a^2}$

(b) Solve the equation $\sin^2\theta + 2\cos^2\theta = 4\sin\theta + 3$ for $0° < \theta < 360°$. [5]

$$\sin^2\theta + 2\cos^2\theta = 4\sin\theta + 3$$
$$\sin^2\theta + 2(1-\sin^2\theta) = 4\sin\theta + 3$$
$$\sin^2\theta + 2 - 2\sin^2\theta = 4\sin\theta + 3$$
$$-\sin^2\theta + 2 = 4\sin\theta + 3$$
$$-\sin^2\theta - 4\sin\theta + 2 - 3 = 0$$
$$-\sin^2\theta - 4\sin\theta - 1 = 0 \quad \text{(Multiply by } -1\text{)}$$
$$\sin^2\theta + 4\sin\theta + 1 = 0$$

Let $y = \sin\theta$

$$y^2 + 4y + 1 = 0$$
$$y = \frac{-4 \pm \sqrt{4^2 - 4\times1\times1}}{2\times1}$$
$$y = \frac{-4 \pm \sqrt{12}}{2}$$
$$y = \frac{-4 \pm 2\sqrt{3}}{2}$$
$$y = \cancel{2}\left[\frac{-2 \pm \sqrt{3}}{\cancel{2}}\right]$$
$$y = -2 \pm \sqrt{3}$$

$y = -2 - \sqrt{3}$

$\sin\theta = -2 - \sqrt{3}$

(NOT DEFINED)

$\sin\theta$ value does not go beyond -1.

$y = -2 + \sqrt{3}$

$\sin\theta = -2 + \sqrt{3} = -0.2679$

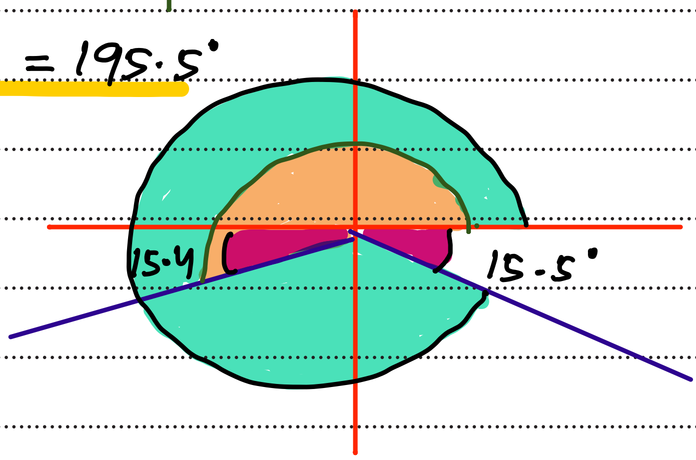
$\theta = \sin^{-1}(-2 + \sqrt{3})$

$\theta = -15.54°$

$\theta = 180 + 15.5 = 195.5°$

$\theta = 360 - 15.5°$

$\theta = 344.5°$



9 The equation of a curve is $y = 4 + 5x + 6x^2 - 3x^3$.

(a) Find the set of values of x for which y decreases as x increases. [4]

If y decreases as x increases, then it is known as decreasing function.

which means $\frac{dy}{dx} < 0$

$$y = 4 + 5x + 6x^2 - 3x^3$$

$$\frac{dy}{dx} = 5 + 12x - 9x^2$$

$$\frac{dy}{dx} < 0$$

$$-9x^2 + 12x + 5 < 0$$

Multiply (-1) on both sides

Sign of inequality changes

$$9x^2 - 12x - 5 > 0$$

$$(3x+1)(3x-5) > 0$$

Coefficient of x^2 is positive. U-shape curve is formed.

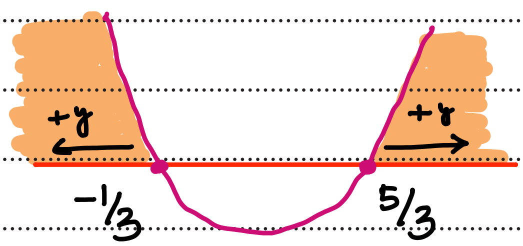
$3x + 1 = 0$

$x = -1/3$

$3x - 5 = 0$

$x = 5/3$

Critical values



$$\boxed{x < \frac{-1}{3} \quad \text{or} \quad x > \frac{5}{3}}$$

(b) It is given that $y = 9x + k$ is a tangent to the curve.

Find the value of the constant k. [4]

$y = 9x + K$

$\frac{dy}{dx} = 9$

Gradient of tangent

$m_{tangent} = 9$

$y = 4 + 5x + 6x^2 - 3x^3$

$\frac{dy}{dx} = 5 + 12x - 9x^2$

Gradient of Curve

$m_{curve} = 5 + 12x - 9x^2$

At the point of intersection of tangent and Curve

$m_{curve} = m_{tangent}$

$5 + 12x - 9x^2 = 9$

$9x^2 - 12x + 9 - 5 = 0$

$9x^2 - 12x + 4 = 0$

$(3x-2)(3x-2) = 0$

$3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

$y = 4 + 5x + 6x^2 - 3x^3$

as $x = \frac{2}{3}$ $\quad y = 4 + 5\left(\frac{2}{3}\right) + 6\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right)^3$

$y = \frac{82}{9}$

Again $\quad y = 9x + K$

$\frac{82}{9} = 9\left(\frac{2}{3}\right) + K$

$\frac{82}{9} = \frac{18}{3} + K$

$K = \frac{82}{9} - 6$

$\boxed{K = \frac{28}{9}}$

10 An arithmetic progression has first term 5 and common difference d, where $d > 0$. The second, fifth and eleventh terms of the arithmetic progression, in that order, are the first three terms of a geometric progression.

(a) Find the value of d. [3]

Arithmetic Progression

$T_n = a + (n-1)d$

$T_2 = 5 + (2-1)d = 5 + d$

$T_5 = 5 + (5-1)d = 5 + 4d$

$T_{11} = 5 + (10-1)d = 5 + 10d$

Geometric Progression

$T_n = ar^{n-1}$

$T_1 = ar^{1-1} = a$

$T_2 = ar^{2-1} = ar$

$T_3 = ar^{3-1} = ar^2$

$5 + d = a$

$5 + 4d = ar$

$5 + 10d = ar^2$

In G.P

Common ratio:

$$\frac{3^{rd}\text{ term}}{2^{nd}\text{ term}} = \frac{2^{nd}\text{ term}}{1^{st}\text{ term}}$$

$$\frac{5+10d}{5+4d} = \frac{5+4d}{5+d}$$

$(5+10d)(5+d) = (5+4d)(5+4d)$

$25 + 5d + 50d + 10d^2 = 25 + 20d + 20d + 16d^2$

$25 + 55d + 10d^2 = 25 + 40d + 16d^2$

$16d^2 - 10d^2 + 40d - 55d + 25 - 25 = 0$

$6d^2 - 15d = 0$

$3d(2d - 5) = 0$

$d = 0$ (NOT POSSIBLE) $\rightarrow$ $d = \frac{5}{2}$

$\boxed{d = 2.5}$

(b) The sum of the first 77 terms of the arithmetic progression is denoted by S_{77}. The sum of the first 10 terms of the geometric progression is denoted by G_{10}.

Find the value of $S_{77} - G_{10}$. [5]

$S_n = \frac{n}{2}[2a + (n-1)d]$ $n = 77,\ a = 5,\ d = 2.5$

$S_{77} = \frac{77}{2}[2 \times 5 + (77-1)2.5]$

$S_{77} = \frac{77}{2}[10 + 76 \times 2.5]$

$S_{77} = 7700$

$G_n = \frac{a(1-r^n)}{1-r}$ $n = 10,\ a = 7.5,\ r = 2$

$G_{10} = \frac{7.5(1-2^{10})}{1-2}$

$G_{10} = 7672.5$

$S_{77} - G_{10}$

$7700 - 7672.5$

$= \boxed{27.5}$

$5 + d = a$

$5 + 2.5 = a$

$a = 7.5$

$ar = 5 + 4d$

$r = \frac{5+4d}{7.5}$

$r = \frac{5+4(2.5)}{7.5}$

$r = \frac{15}{7.5}$

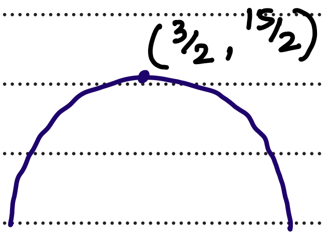
$r = 2$

11 The function f is defined by $f(x) = 3 + 6x - 2x^2$ for $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants, and state the range of f. [3]

$$f(x) = 3 + 6x - 2x^2$$
$$f(x) = -2x^2 + 6x - 3$$
$$= -2\left[x^2 - 3x - \frac{3}{2}\right]$$
$$= -2\left[\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - \frac{3}{2}\right]$$
$$= -2\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{3}{2}\right]$$
$$= -2\left[\left(x - \frac{3}{2}\right)^2 - \frac{15}{4}\right]$$
$$= \boxed{\frac{15}{2} - 2\left(x - \frac{3}{2}\right)^2}$$

Max. value of y: $\frac{15}{2}$

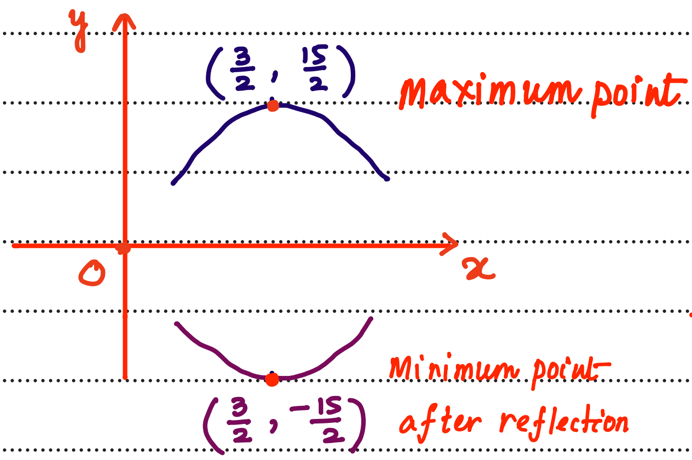


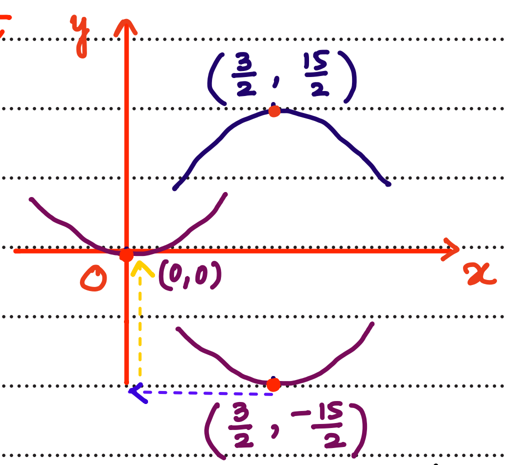
Range of f(x)

$$\boxed{f(x) \le \frac{15}{2}}$$

(b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin.

Give details of the reflection and translation involved. [2]





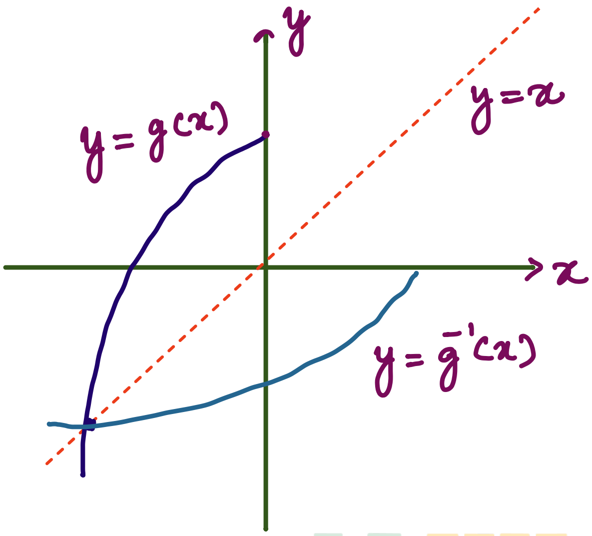
(i) Reflection in x-axis

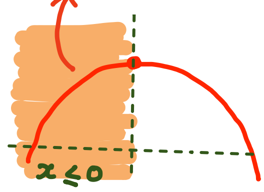
(ii) Translation by $\begin{pmatrix} -3/2 \\ 15/2 \end{pmatrix}$

The function g is defined by $g(x) = 3 + 6x - 2x^2$ for $x \leqslant 0$.

Will be considered one side of the graph.

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]





The coefficient of x^2 is negative, n-shaped curve is expected.

Each value of y is associated with one value of x.

So g(x) is one to one function $x \leq 0$.

(d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in **(c)**, and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

$$g(x) = \frac{15}{2} - 2\left(x - \frac{3}{2}\right)^2$$

$$y = \frac{15}{2} - 2\left(x - \frac{3}{2}\right)^2$$

$y = g^{-1}(x)$ is reflected in the line $y = x$.

$$2\left(x - \frac{3}{2}\right)^2 = \frac{15}{2} - y$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{1}{2}\left(\frac{15}{2} - y\right)$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{15}{4} - \frac{y}{2}$$

$$x - \frac{3}{2} = \sqrt{\frac{15}{4} - \frac{y}{2}}$$

$$x = \frac{3}{2} + \sqrt{\frac{15}{4} - \frac{y}{2}}$$

$$f^{-1}(x) = \frac{3}{2} + \sqrt{\frac{15}{4} - \frac{x}{2}}$$

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE