1. Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

The equation of a curve is $y = \dfrac{e^{\sin x}}{\cos^2 x}$ for $0 \leqslant x \leqslant 2\pi$.

Find $\dfrac{dy}{dx}$ and hence find the x-coordinates of the stationary points of the curve. [7]

(a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec}\frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α, in the interval $0 < x < \pi$. [2]

(b) Verify by calculation that α lies between 1 and 2. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln(\operatorname{cosec} \tfrac{1}{2}x_n + 3)$$

converges, then it converges to α. [1]

(d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

5.

Use the substitution $u = 1 - \sin x$ to find the exact value of

$$\int_{\pi}^{\frac{3}{2}\pi} \frac{\sin 2x}{\sqrt{1-\sin x}} \mathrm{d}x.$$

Give your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

6.

(a) Given that $2x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$. [3]

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x\tan^{-1}(2x)\,dx$. [7]

7.

Express $\frac{6x^2-9x-16}{2x^2-5x-12}$ in partial fractions. [5]

8. The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]

9.

The equation of a curve is $ye^{2x} + y^2e^x = 6$.

Find the gradient of the curve at the point where $y = 1$. [6]

(a) It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

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(b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(5 + \cos 3x_n\right)$$

converges then it converges to the root of the equation in part **(a)**. [1]

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(c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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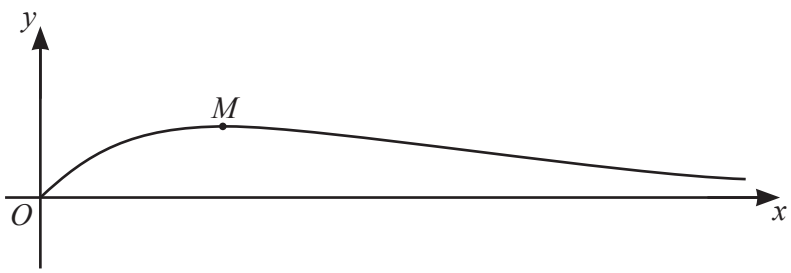
11.



The diagram shows the curve $y = xe^{-ax}$, where a is a positive constant, and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the exact value of $\int_0^{\frac{2}{a}} xe^{-ax}\,dx$. [5]

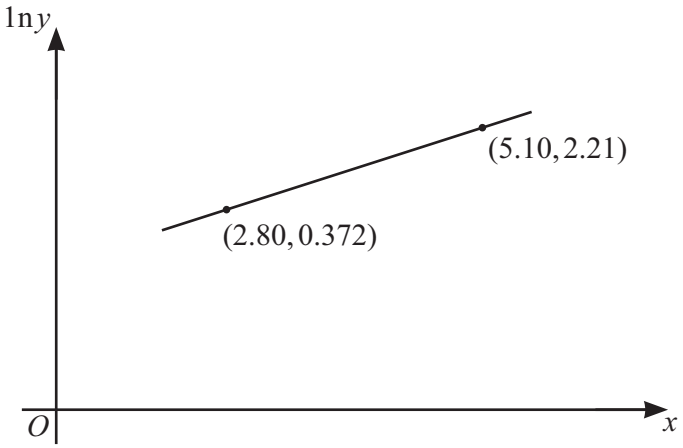
12.

(a) Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} \left(\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta\right) \mathrm{d}\theta$. [6]

Solve the equation $8^{3-6x} = 4 \times 5^{-2x}$. Give your answer correct to 3 decimal places. [4]

14.



The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c. Give each value correct to 2 significant figures. [4]

(a) Express $3\cos 2x - \sqrt{3}\sin 2x$ in the form $R\cos(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{\left(3\cos 2x - \sqrt{3}\sin 2x\right)^2}\,\mathrm{d}x$, simplifying your answer. [5]

16.

Use the substitution $2x = \tan\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{12}{\left(1+4x^2\right)^2}\,\mathrm{d}x\ .$$

Give your answer in the form $a+b\pi$, where a and b are rational numbers. [9]