

**1.**

It is given that  $x = \ln(2y - 3) - \ln(y + 4)$ .

Express  $y$  in terms of  $x$ .

[3]



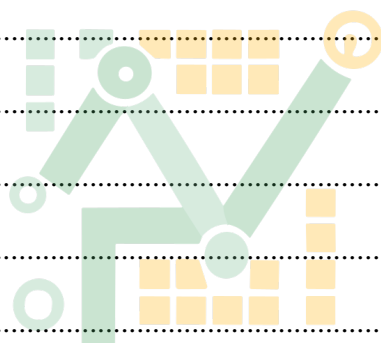
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**2.**

The polynomial  $2x^4 + ax^3 + bx - 1$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . When  $p(x)$  is divided by  $x^2 - x + 1$  the remainder is  $3x + 2$ .

Find the values of  $a$  and  $b$ .

[5]



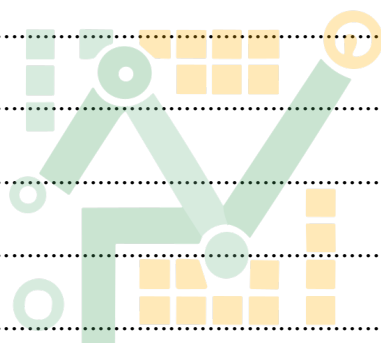
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The parametric equations of a curve are

(a) Show that  $\frac{dy}{dx} = e^{-2t}$ . [3]

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[3]



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- (b) Hence show that the normal to the curve, where  $t = -1$ , passes through the point  $\left(0, 3 - \frac{1}{e^4}\right)$ . [3]



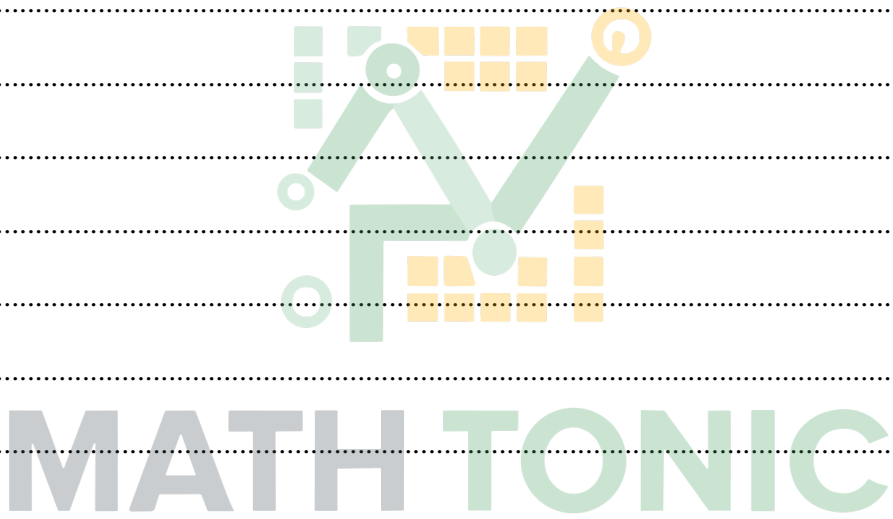
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**4.**

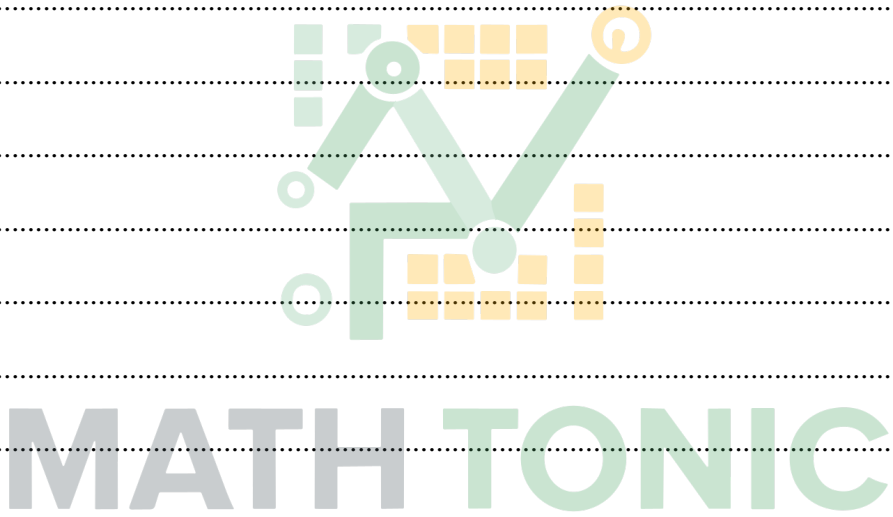
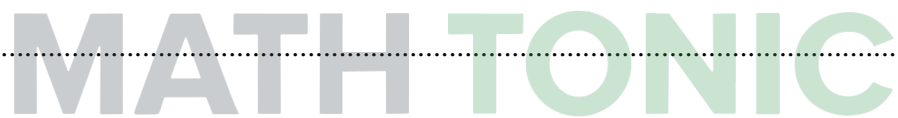
- (a) Express  $5 \sin \theta + 12 \cos \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]



# MATH TONIC



MATH TONIC

The logo for 'Math Tonic' is centered on the page. It features a stylized green letter 'M' composed of thick lines. Small yellow squares and circles are placed at various points along the lines of the 'M'. Below the graphic, the words 'MATH TONIC' are written in a bold, sans-serif font. 'MATH' is in grey and 'TONIC' is in green.

(a) Find the exact coordinates of  $M$ .

[4]



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- (b)** Find the exact area of the shaded region bounded by the curve, the  $x$ -axis and the line  $x = \frac{1}{2}$ . [5]



# MATH TONIC

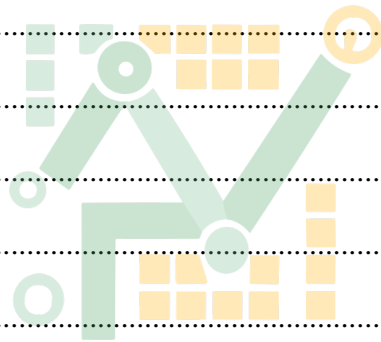


**6.**

Let  $f(x) = \frac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$ .

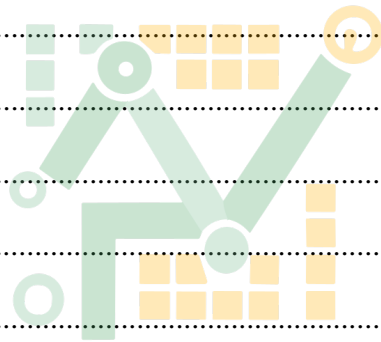
(a) Express  $f(x)$  in partial fractions.

[5]



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[5]

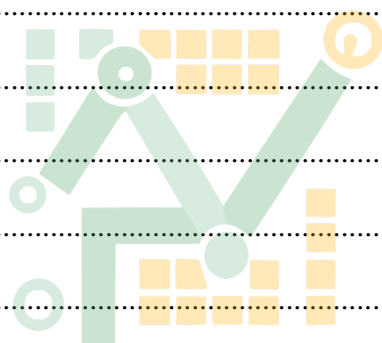


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**7.**

Find the coefficient of  $x^3$  in the binomial expansion of  $(3 + x)\sqrt{1 + 4x}$ .

[4]



# MATH TONIC


**8.**

- (a) Show that the equation  $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$  can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0. \quad [2]$$

[illegible]

- (b) Hence solve the equation  $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$  for  $0^\circ < \theta < 180^\circ$ . [4]

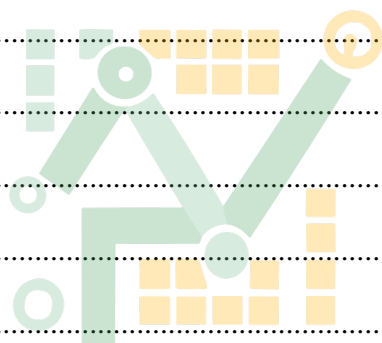


# MATH TONIC

**9.**

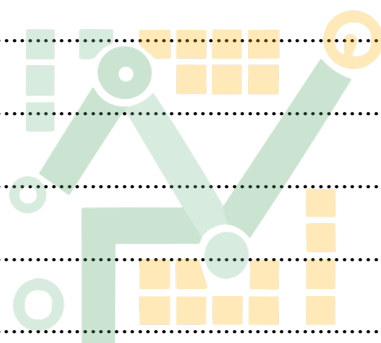
The equation of a curve is  $x^2y - ay^2 = 4a^3$ , where  $a$  is a non-zero constant.

(a) Show that  $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ . [4]



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- (b)** Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]



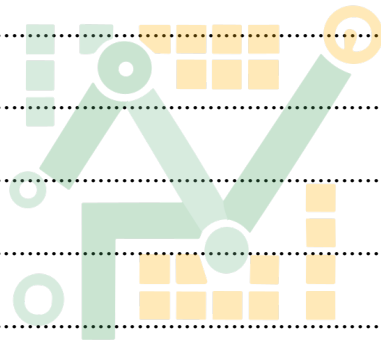
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**10.**

The constant  $a$  is such that  $\int_0^a x e^{-2x} \, dx = \frac{1}{8}$ .

(a) Show that  $a = \frac{1}{2} \ln(4a + 2)$ .

[5]



# MATH TONIC

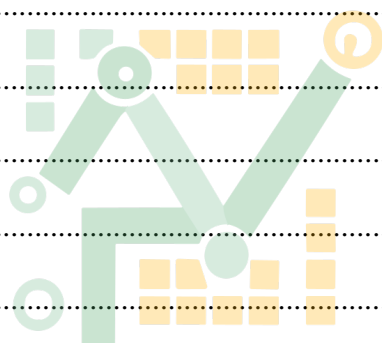




**11.**

Solve the inequality  $|5x - 3| < 2|3x - 7|$ .

[4]



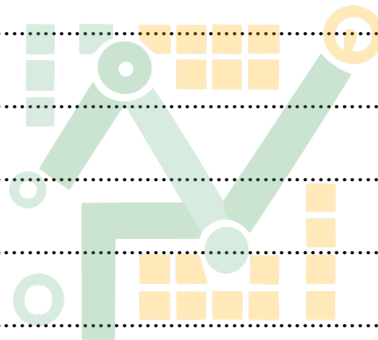
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**12.**

The equation of a curve is  $3x^2 + 4xy + 3y^2 = 5$ .

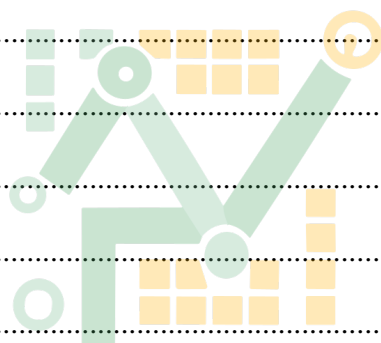
(a) Show that  $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$ .

[4]



# MATH TONIC

- (b)** Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to  $y + 2x = 0$ . [5]



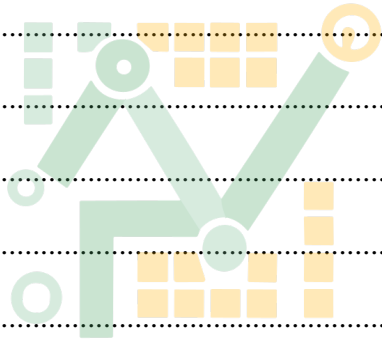
# MATH TONIC

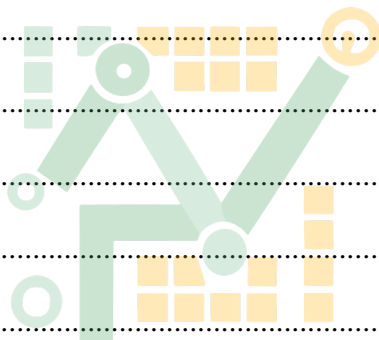
(a) Find the exact coordinates of  $M$ .

[5]



# MATH TONIC

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- # MATH TONIC



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