1.

It is given that $x = \ln(2y - 3) - \ln(y + 4)$.

Express y in terms of x. [3]

$$x = \ln(2y-3) - \ln(y+4)$$

$$x = \ln\left(\frac{2y-3}{y+4}\right)$$

$$e^x = \frac{2y-3}{y+4}$$

$$e^x(y+4) = 2y-3$$

$$ye^x + 4e^x = 2y-3$$

$$ye^x - 2y = -4e^x - 3$$

$$y(e^x - 2) = -4e^x - 3$$

$$y = \frac{-4e^x-3}{e^x-2}$$

$$y = \frac{-(4e^x+3)}{e^x-2}$$

$$y = \frac{4e^x+3}{2-e^x}$$

logarithm Rules:

$\ln a - \ln b = \ln \frac{a}{b}$

If $x = \ln y$

$e^x = y$

2.

The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b. [5]

$$
\begin{array}{r|l}
 & 2x^2 + (a+2)x + a \\
\hline
x^2 - x + 1 & 2x^4 + ax^3 \quad bx - 1 \\
 & 2x^4 - 2x^3 + 2x^2 \\
\hline
 & (a+2)x^3 - 2x^2 + bx - 1 \\
 & (a+2)x^3 - (a+2)x^2 + (a+2)x \\
\hline
 & (a+2-2)x^2 + (b-a-2)x - 1 \\
 & ax^2 + (b-a-2)x - 1 \\
 & ax^2 - ax + a \\
\hline
 & (b-a-2+a)x - a - 1 \\
 & (b-2)x - (a+1)
\end{array}
$$

Remainder

Remainder $= 3x + 2$

$(b-2)x - (a+1) = 3x + 2$

Comparing coefficients

$b - 2 = 3$

$\boxed{b = 5}$

$-(a+1) = 2$

$a + 1 = -2$

$a = -2 - 1$

$\boxed{a = -3}$

The parametric equations of a curve are

$$x = te^{2t}, \qquad y = t^2 + t + 3.$$

(a) Show that $\frac{dy}{dx} = e^{-2t}$. [3]

$x = te^{2t}$

$\frac{dx}{dt} = t\frac{d}{dt}(e^{2t}) + e^{2t}\frac{d}{dt}(t)$

$\frac{dx}{dt} = t(2e^{2t}) + e^{2t}(1)$

$\frac{dx}{dt} = 2te^{2t} + e^{2t}$

$\frac{dx}{dt} = e^{2t}(2t+1)$

$\frac{dt}{dx} = \frac{1}{e^{2t}(2t+1)}$

$y = t^2 + t + 3$

$\frac{dy}{dt} = 2t + 1$

Rule:

$\frac{d}{dx}(uv) = uv' + vu'$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \cancel{(2t+1)} \times \frac{1}{e^{2t}\cancel{(2t+1)}}$

$= \frac{1}{e^{2t}}$

$\boxed{\frac{dy}{dx} = e^{-2t}}$

(b) Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0,\ 3 - \frac{1}{e^4}\right)$. [3]

$$\frac{dy}{dx} = e^{-2t}$$

at $t = -1$, $\frac{dy}{dx} = e^{-2(-1)}$

$$\frac{dy}{dx} = e^{2}$$

Gradient of Curve $= e^2$ (m_{curve})

for perpendicular $m_1 \times m_2 = -1$

Gradient of Normal $= \frac{-1}{e^2}$ (m_{normal})

Finding the Coordinates

$t = -1$

$x = te^{2t}$ | $y = t^2 + t + 3$

$x = -1 \times e^{-2}$ | $y = (-1)^2 + (-1) + 3$

$x = -e^{-2}$ | $y = 1 - 1 + 3$

$x = \frac{-1}{e^2}$ | $y = 3$

Coordinates $\left(-\frac{1}{e^2}, 3\right)$

Equation of Normal

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{e^2}\left(x - \left(-\frac{1}{e^2}\right)\right)$

$y - 3 = \frac{-1}{e^2}\left(x + \frac{1}{e^2}\right)$

$y - 3 = \frac{-x}{e^2} - \frac{1}{e^4}$

Substitute $x = 0$

$y - 3 = 0 - \frac{1}{e^4}$

$y = 3 - \frac{1}{e^4}$

Yes, it passes through $\left(0,\ 3 - \frac{1}{e^4}\right)$

4.

(a) Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

$5\sin\theta + 12\cos\theta = R\cos(\theta - \alpha)$

$5\sin\theta + 12\cos\theta = R[\cos\theta\cdot\cos\alpha + \sin\theta\sin\alpha]$

$5\sin\theta + 12\cos\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

Comparing the Coefficients :

$R\sin\alpha = 5$ —— (i) $\quad R\cos\alpha = 12$ —— (ii)

Divide (i) by (ii)

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{5}{12}$

Formula:

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan\alpha = \frac{5}{12}$

$\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\alpha = 0.395$

$R\sin\alpha = 5 \quad R\cos\alpha = 12$

$R^2\sin^2\alpha = 25$ —— (iii) $\quad R^2\cos^2\alpha = 144$ —— (iv)

Adding (iii) and (iv)

$R^2\sin^2\alpha + R^2\cos^2\alpha = 25 + 144$

$R^2(\sin^2\alpha + \cos^2\alpha) = 169$

$R^2 \times 1 = \sqrt{169}$

$R = 13$

$\boxed{R\cos(\theta - \alpha) = 13\cos(\theta - 0.395)}$

(b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$ for $0 \leqslant x \leqslant \pi$. [4]

$5\sin\theta + 12\cos\theta = 13\cos(\theta - 0.395)$

$5\sin 2x + 12\cos 2x = 13\cos(2x - 0.395)$

$6 = 13\cos(2x - 0.395)$

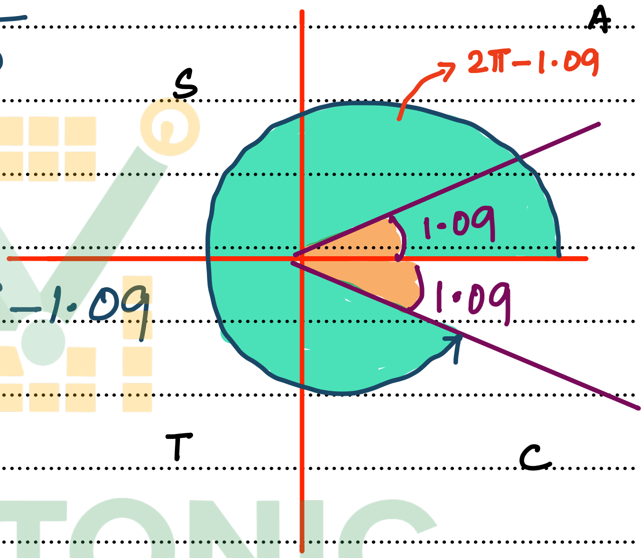
$\cos(2x - 0.395) = \frac{6}{13}$

let $y = 2x - 0.395$

$\cos y = \frac{6}{13}$

$y = \cos^{-1}\left(\frac{6}{13}\right)$

$y = 1.09;\quad 2\pi - 1.09$



$y = 1.09$

$2x - 0.395 = 1.09$

$2x = 1.09 + 0.395$

$x = \frac{1.09 + 0.395}{2}$

$x = 0.743$

$y = 2\pi - 1.09$

$2x - 0.395 = 2\pi - 1.09$

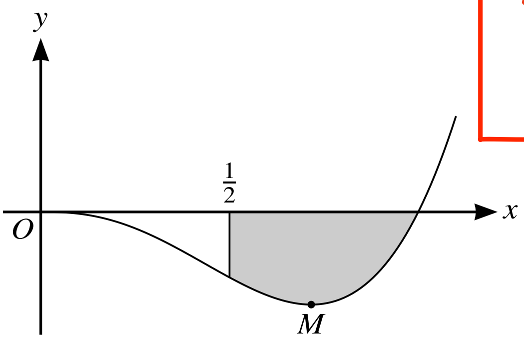
$2x = 2\pi - 1.09 + 0.395$

$x = \frac{2\pi - 1.09 + 0.395}{2}$

$x = 2.79$

5.

Rule : $\frac{d}{dx}(uv) = uv' + vu'$



The diagram shows the curve $y = x^3 \ln x$, for $x > 0$, and its minimum point M.

(a) Find the exact coordinates of M. [4]

At Stationary point $\frac{dy}{dx} = 0$

$$y = x^3 \ln x$$

$$\frac{dy}{dx} = x^3 \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x^3)$$

$$\frac{dy}{dx} = x^3 \times \frac{1}{x} + \ln x \times 3x^2$$

$$\frac{dy}{dx} = x^2 + 3x^2 \ln x$$

$$0 = x^2(1 + 3\ln x)$$

$x^2 = 0$ $\qquad$ $1 + 3\ln x = 0$

$x = 0$ $\qquad$ $3\ln x = -1$

NOT NEEDED $\qquad$ $\ln x = \frac{-1}{3}$

$$x = e^{-1/3}$$

Find the value of y : $y = x^3 \ln x$

$$y = (e^{-1/3})^3 \ln e^{-1/3}$$

$$y = e^{-1} \times \left(\frac{-1}{3}\right) \times \ln e$$

$$y = -\frac{1}{3} e^{-1}$$

$$\boxed{\left(e^{-1/3}, \frac{-1}{3} e^{-1}\right)}$$

(b) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = \frac{1}{2}$. [5]

$y = 0$

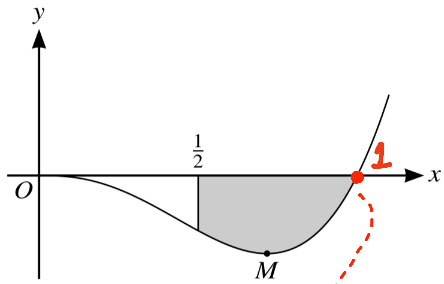
$x^3 \ln x = 0$

$x^3 = 0 \qquad \ln x = 0$

$x = 0 \qquad x = e^0$

NOT NEEDED

$x = 1$



To find this x-coordinate
Put $y = 0$

To find the area under curve we need to do integration of curve.

$$\int_{\frac{1}{2}}^{1} x^3 \underbrace{\ln x}_{u} . dx$$

dv

I L A T E

Logarithmic

Algebraic

$u = \ln x \qquad dv = x^3 . dx$

$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int x^3 . dx$

$v = \frac{x^4}{4}$

$du = \frac{dx}{x}$

Integration by parts Rule

$\boxed{u . dv = uv - \int v . du}$

$$\int x^3 \ln x \cdot dx = \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \times \frac{1}{x} dx$$

$$= \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^3 . dx$$

$$= \frac{1}{4} x^4 \ln x - \frac{1}{4} \times \frac{x^4}{4}$$

$$= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4$$

$$\int_{\frac{1}{2}}^{1} x^3 \ln x \cdot dx = \left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4\right]_{\frac{1}{2}}^{1}$$

$$= \left(\frac{1}{4}(1)^4 \ln 1 - \frac{1}{16}(1)^4\right) - \left(\frac{1}{4}\left(\frac{1}{2}\right)^4 \ln\left(\frac{1}{2}\right) - \frac{1}{16}\left(\frac{1}{2}\right)^4\right)$$

$\ln 1 = 0$

$$= \left(0 - \frac{1}{16}\right) - \left(\frac{1}{64}\ln\left(\frac{1}{2}\right) - \frac{1}{256}\right)$$

$$= -\frac{1}{16} - \frac{1}{64}\ln\left(\frac{1}{2}\right) + \frac{1}{256}$$

$$= \frac{-15}{256} - \frac{1}{64}\ln 2^{-1}$$

$$= \frac{-15}{256} - (-1)\frac{1}{64}\ln 2$$

$$= \left[\frac{-15}{256} + \frac{1}{64}\ln 2\right] \times (-1) \quad \boxed{= \frac{15}{256} - \frac{1}{64}\ln 2}$$

This is negative because it is below x-axis

But Area Can't be negative, To make it positive, multiply the expression by (-1).

6.

Let $\mathrm{f}(x) = \dfrac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express f(x) in partial fractions. [5]

$$\frac{5x^2+x+11}{(4+x^2)(1+x)} = \frac{Ax+B}{4+x^2} + \frac{C}{1+x}$$

$$= \frac{(Ax+B)(1+x) + C(4+x^2)}{(4+x^2)(1+x)}$$

$$5x^2+x+11 = (Ax+B)(1+x) + C(4+x^2)$$

$x=-1$, $5(-1)^2-1+11 = 0 + C(4+(-1)^2)$

$15 = 5C$

$C = 3$

$x=0$, $5(0)^2+0+11 = (A(0)+B)(1+0)+C(4+0)$

$11 = B + 4C$

$11 = B + 4\times 3$

$11 = B + 12$

$B = -1$

$x=1$, $5(1)^2+1+11 = (A(1)+B)(1+1)+C(4+1^2)$

$17 = 2(A+B) + 5C$

$17 = 2(A-1) + 5\times 3$

$17-15 = 2(A-1)$

$2 = 2(A-1)$

$1 = A-1 \Rightarrow A = 2$

$$\boxed{\frac{5x^2+x+11}{(4+x^2)(1+x)} = \frac{2x-1}{4+x^2} + \frac{3}{1+x}}$$

(b) Hence show that $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = \ln 54 - \frac{1}{8}\pi$. [5]

$$\int_0^2 \left(\frac{2x-1}{4+x^2} + \frac{3}{1+x}\right) dx$$

$$\int_0^2 \left(\frac{2x}{4+x^2} - \frac{1}{4+x^2} + \frac{3}{1+x}\right) dx$$

$$\int_0^2 \left(\frac{2x}{4+x^2} - \frac{1}{2^2+x^2} + \frac{3}{1+x}\right) dx$$

$$\boxed{\int \frac{\square'}{\square}\cdot dx = \ln \square} \quad \boxed{\int \frac{1}{a^2+x^2}\cdot dx = \frac{1}{a}\tan^{-1}\frac{x}{a}} \quad \boxed{\int \frac{1}{x}\cdot dx = \ln x}$$

$$\left[\ln|4+x^2| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 3\ln|1+x|\right]_0^2$$

$$\left(\ln|4+2^2| - \frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right) + 3\ln|1+2|\right) - \left(\ln|4+0| - \frac{1}{2}\tan^{-1}(0) + 3\ln(1+0)\right)$$

$$\ln 8 - \frac{1}{2}\times\frac{\pi}{4} + 3\ln 3 - (\ln 4 - 0 + 3\ln 1)$$

$\ln 1 = 0$

$$\ln 8 - \frac{\pi}{8} + \ln 3^3 - \ln 4$$

$$\ln 8 + \ln 27 - \ln 4 - \frac{\pi}{8}$$

$$\ln\left[\frac{8\times 27}{4}\right] - \frac{\pi}{8} = \boxed{\ln 54 - \frac{\pi}{8}}$$

7.

Find the coefficient of x^3 in the binomial expansion of $(3+x)\sqrt{1+4x}$. [4]

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

$\sqrt{1+4x}$

$$(1+4x)^{\frac{1}{2}} = 1 + \frac{1}{2}(4x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(4x)^2 + \frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(4x)^3$$

$$= 1 + 2x - \frac{1}{8} \times 16x^2 + \frac{1}{16} \times 64x^3$$

$$= 1 + 2x - 2x^2 + 4x^3$$

$$(3+x)\sqrt{1+4x} = (3+x)(1+2x-2x^2+4x^3)$$

Finding x^3 terms:

$$3 \times 4x^3 = 12x^3$$

$$x \times (-2x^2) = -2x^3$$

$$10x^3$$

Coefficient of $x^3 = 10$

8.

(a) Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0. \qquad [2]$$

$\sin 2\theta + \cos 2\theta = 2\sin^2\theta$

$2\sin\theta \cdot \cos\theta + \cos^2\theta - \sin^2\theta = 2\sin^2\theta$

$2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta - 2\sin^2\theta = 0$

$2\sin\theta\cos\theta + \cos^2\theta - 3\sin^2\theta = 0$

$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$

Formula: $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

$\sin 2\theta + \cos 2\theta = 2\sin^2\theta$

$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$

$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$

$\cos^2\theta + 3\sin\theta \cdot \cos\theta - 1\sin\theta \cdot \cos\theta - 3\sin^2\theta = 0$

$\cos\theta(\cos\theta + 3\sin\theta) - \sin\theta(\cos\theta + 3\sin\theta) = 0$

$(\cos\theta + 3\sin\theta)(\cos\theta - \sin\theta) = 0$

$\cos\theta - \sin\theta = 0$

$\cos\theta = \sin\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

$\cos\theta + 3\sin\theta = 0$

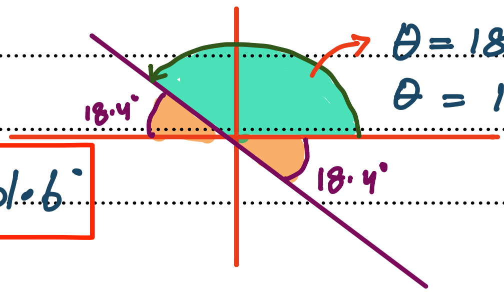
$3\sin\theta = -\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{-1}{3}$

$\tan\theta = \frac{-1}{3}$

$\theta = \tan^{-1}(-1/3)$

$\theta = -18.4°$



$\theta = 180 - 18.4°$

$\theta = 161.4°$

$\boxed{\theta = 45°, 161.6°}$

9.

The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

$$x^2y - ay^2 = 4a^3$$

$$\frac{d}{dx}(x^2y) - \frac{d}{dx}(ay^2) = \frac{d}{dx}(4a^3)$$

$$x^2\frac{dy}{dx} + y\frac{d}{dx}(x^2) - a\frac{d}{dx}(y^2) = 0$$

$$x^2\frac{dy}{dx} + 2xy - 2ay\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} - 2ay\frac{dy}{dx} + 2xy = 0$$

$$\frac{dy}{dx}(x^2 - 2ay) = -2xy$$

Rule: $\frac{d}{dx}(uv) = uv' + vu'$

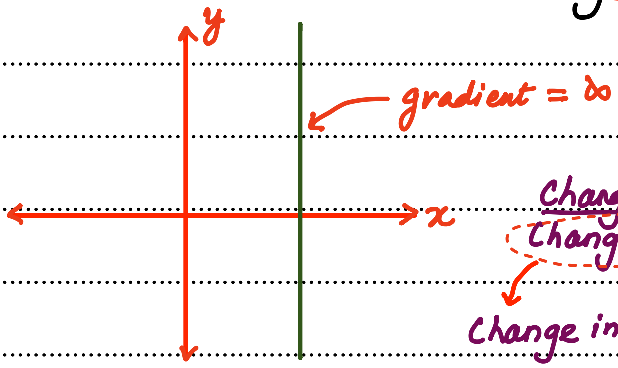
$$\frac{dy}{dx} = \frac{-2xy}{x^2 - 2ay}$$

Multiply by (-1) in Numerator and Denominator

$$\boxed{\frac{dy}{dx} = \frac{2xy}{2ay - x^2}}$$

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

when tangent to the curve is parallel to y-axis which means gradient $= \infty$



$\dfrac{\text{change in } y}{\text{change in } x} = \text{gradient}$

$\dfrac{y_2 - y_1}{0} = \infty$

change in $x = 0$ when a line is parallel to y-axis

$$\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$$

Denominator $= 0$

$$2ay - x^2 = 0$$

$$x^2 = 2ay$$

$$x^2 y - ay^2 = 4a^3$$

$$(2ay)y - ay^2 = 4a^3$$

$$2ay^2 - ay^2 = 4a^3$$

$$ay^2 = 4a^3$$

$$y^2 = 4a^2$$

$$y = \sqrt{4a^2}$$

$$y = \pm 2a$$

$y = 2a$

$x^2 = 2ay$

$x^2 = 2a(2a)$

$x^2 = 4a^2$

$x = \pm 2a$

$y = -2a$

$x^2 = 2ay$

$x^2 = 2a(-2a) = -4a^2$

NO SOLUTIONS because square can't be negative.

$\boxed{(2a, 2a) \text{ and } (-2a, -2a)}$

10.

The constant a is such that $\int_0^a xe^{-2x}\,dx = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2}\ln(4a+2)$. [5]

Exponential

I L A T E

Algebraic

$$\int_0^a \underbrace{x}_{u}\,\overbrace{e^{-2x}}^{dv}\,dx$$

$$\boxed{\begin{array}{c}\text{Integration by parts Rule}\\ u\,dv = uv - \int v.du\end{array}}$$

$u = x$ $\qquad dv = e^{-2x}dx$

$\frac{du}{dx} = 1$ $\qquad \int dv = \int e^{-2x}.dx$

$\qquad V = -\frac{1}{2}e^{-2x}$

$du = dx$

$$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}.dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}.dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\times\frac{e^{-2x}}{-2}$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$$

$$\int_0^a xe^{-2x}.dx = \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^a$$

$$= \left(-\frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a}\right) - \left(0 - \frac{1}{4}e^{0}\right)$$

$$= -\frac{a}{2}e^{-2a} - \frac{1}{4}e^{-2a} + \frac{1}{4}$$

$$\int_0^a x e^{-2x} \cdot dx = \frac{1}{8}$$

$$-\frac{a}{2}e^{-2a} - \frac{1}{4}e^{-2a} + \frac{1}{4} = \frac{1}{8}$$

multiply 8 with each term.

$$-4ae^{-2a} - e^{-2a} + 2 = 1$$

$$-4ae^{-2a} - e^{-2a} + 1 = 0$$

multiply (-1) with each term

$$4ae^{-2a} + e^{-2a} - 1 = 0$$

$$4ae^{-2a} + e^{-2a} = 1$$

$$e^{-2a}(4a+2) = 1$$

$$e^{-2a} = \frac{1}{4a+2}$$

$$\frac{1}{e^{2a}} = \frac{1}{4a+2}$$

$$e^{2a} = 4a+2$$

$$\ln e^{2a} = \ln(4a+2)$$

$$2a \ln e = \ln(4a+2)$$

$\ln e = 1$

$$2a = \ln(4a+2)$$

$$\boxed{a = \frac{1}{2}\ln(4a+2)}$$

(b) Verify by calculation that a lies between 0.5 and 1. [2]

$$a = \frac{1}{2}\ln(4a+2)$$

$$a - \frac{1}{2}\ln(4a+2) = 0$$

$$f(a) = a - \frac{1}{2}\ln(4a+2)$$

Put $a = 0.5$, $0.5 - \frac{1}{2}\ln(4\times 0.5 + 2) = -0.193$

$f(0.5) < 0$

Put $a = 1$, $1 - \frac{1}{2}\ln(4\times 1 + 2) = 0.104$

$f(1) > 0$

There is a sign change between $f(0.5)$ and $f(1)$

so, a lies between 0.5 and 1.

(c) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$$a_{n+1} = \frac{1}{2}\ln(4a_n+2)$$

Initial value $a_0 = 0.5$

$a_1 = \frac{1}{2}\ln(4\times \underset{a_0}{0.5} + 2) = 0.6391$

$a_2 = \frac{1}{2}\ln(4\times \underset{a_1}{0.6391} + 2) = 0.7814$

$a_3 = \frac{1}{2}\ln(4\times \underset{a_2}{0.7814} + 2) = 0.8171$

$a_4 = \frac{1}{2}\ln(4\times \underset{a_3}{0.8171} + 2) = 0.8309$

$a_5 = \frac{1}{2}\ln(4\times \underset{a_4}{0.8309} + 2) = 0.8361$

$a_6 = \frac{1}{2}\ln(4\times \underset{a_5}{0.8361} + 2) = 0.8380$

$a_7 = \frac{1}{2}\ln(4\times \underset{a_6}{0.8380} + 2) = 0.8388$

0.84 (2dp)

$$\boxed{a = 0.84 \text{ (2dp)}}$$

11.

Solve the inequality $|5x-3| < 2|3x-7|$. [4]

$$|5x-3| < 2|3x-7|$$

Squaring both Sides

$$(5x-3)^2 < [2(3x-7)]^2$$

$$(5x-3)^2 < (6x-14)^2$$

$$\underbrace{(5x-3)}_{a}^2 - \underbrace{(6x-14)}_{b}^2 < 0$$

$a^2 - b^2 = (a+b)(a-b)$

$$(\underbrace{5x-3}_{a} \underbrace{+}_{+} \underbrace{6x-14}_{b})(\underbrace{5x-3}_{a} \underbrace{-}_{-} \underbrace{6x+14}_{b}) < 0$$

$$(11x-17)(-x+11) < 0$$

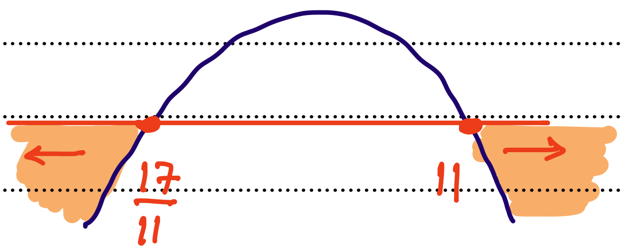
To find critical points:

$11x - 17 = 0$ $\qquad$ $-x+11 = 0$

$x = \frac{17}{11}$ $\qquad$ $x = 11$

As the Coefficient of x^2 is negative, n-shaped Curve is expected.

We have to Consider negative y-values



$$\boxed{x < \frac{17}{11} \quad \text{and} \quad x > 11}$$

12.

The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$. [4]

$$3x^2 + 4xy + 3y^2 = 5$$

$$\frac{d}{dx}(3x^2) + \frac{d}{dx}(4xy) + \frac{d}{dx}(3y^2) = \frac{d}{dx}(5)$$

$$3(2x) + 4x\cdot\frac{dy}{dx} + y(4) + 3(2y)\frac{dy}{dx} = 0$$

$$6x + 4x\cdot\frac{dy}{dx} + 4y + 6y\frac{dy}{dx} = 0$$

$$4x\frac{dy}{dx} + 6y\frac{dy}{dx} = -6x - 4y$$

$$\frac{dy}{dx}(4x+6y) = -6x-4y$$

$$\frac{dy}{dx} = \frac{-6x-4y}{4x+6y}$$

$$\frac{dy}{dx} = \frac{-\cancel{2}(3x+2y)}{\cancel{2}(2x+3y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-(3x+2y)}{2x+3y}}$$

(b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

$y + 2x = 0$

$y = -2x$

gradient of tangent $= -2$

Parallel lines have the same gradient

$\frac{dy}{dx} = -2$

$\frac{-(3x+2y)}{2x+3y} = -2$

$3x + 2y = 2(2x+3y)$

$3x + 2y = 4x + 6y$

$4x - 3x = 2y - 6y$

$x = -4y$

Equation of the curve:

$3x^2 + 4xy + 3y^2 = 5$

Put $x = -4y$, $3(-4y)^2 + 4(-4y)y + 3y^2 = 5$

$48y^2 - 16y^2 + 3y^2 = 5$

$35y^2 = 5$

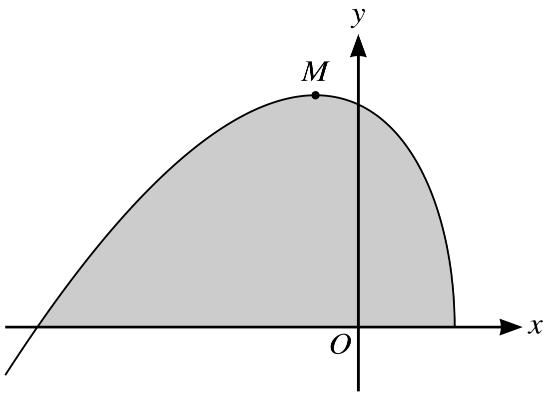
$y^2 = \frac{1}{7}$

$y = \pm\sqrt{\frac{1}{7}} = \pm\frac{1}{\sqrt{7}}$

$y = \frac{1}{\sqrt{7}}$	$y = -\frac{1}{\sqrt{7}}$
$x = -4\left(\frac{1}{\sqrt{7}}\right) = \frac{-4}{\sqrt{7}}$	$x = -4\left(-\frac{1}{\sqrt{7}}\right) = \frac{4}{\sqrt{7}}$

Exact Coordinates $\left(\frac{-4}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right), \left(\frac{4}{\sqrt{7}}, \frac{-1}{\sqrt{7}}\right)$

13.



The diagram shows the curve $y = (x+5)\sqrt{3-2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

At stationary point $\frac{dy}{dx} = 0$

$$y = (x+5)\sqrt{3-2x}$$

$$\frac{dy}{dx} = (x+5)\frac{d}{dx}(3-2x)^{1/2} + (3-2x)^{1/2}\frac{d}{dx}(x+5)$$

$$\frac{dy}{dx} = (x+5)\frac{1}{2}(3-2x)^{-1/2}(-2) + (3-2x)^{1/2}(1)$$

$$\frac{dy}{dx} = -(x+5)(3-2x)^{-1/2} + (3-2x)^{1/2}$$

$$0 = \frac{-(x+5)}{(3-2x)^{1/2}} + (3-2x)^{1/2}$$

Multiplying $(3-2x)^{1/2}$ in each term

$$0 = -(x+5) + (3-2x)$$

$$-x-5+3-2x = 0$$

$$-3x-2 = 0$$

$$-3x = 2$$

$$x = \frac{-2}{3}$$

$$y = (x+5)\sqrt{3-2x}$$

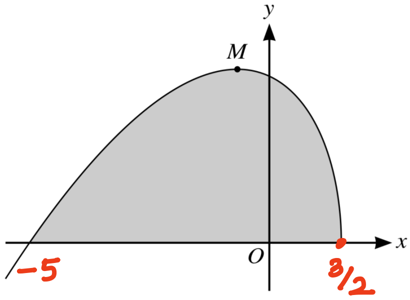
$$y = \left(-\frac{2}{3}+5\right)\left(3-2\left(-\frac{2}{3}\right)\right)^{1/2}$$

$$y = \frac{13}{3} \times \left(\frac{13}{3}\right)^{1/2}$$

$$y = \left(\frac{13}{3}\right)^{3/2}$$

$$\boxed{M\left(-\frac{2}{3}, \left(\frac{13}{3}\right)^{3/2}\right)}$$

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]



$u = 3 - 2x$

$\frac{du}{dx} = -2$

$dx = \frac{du}{-2}$

$u = 3 - 2x$

$2x = 3 - u$

$x = \frac{3-u}{2}$

On x-axis, $y = 0$

$(x+5)\sqrt{3-2x} = 0$

$x + 5 = 0$

$x = -5$

$(3-2x)^{1/2} = 0$

$3 - 2x = 0$

$2x = 3$

$x = \frac{3}{2}$

Change in limits:

$u = 3 - 2x$

$x = 3/2$, $u = 3 - 2 \times 3/2 = 0$

$x = -5$, $u = 3 - 2(-5) = 13$

$$\int_{-5}^{3/2} (x+5)\sqrt{3-2x} \cdot dx$$

$$\int_{13}^{0} \left(\frac{3-u}{2} + 5\right)\sqrt{u}\, \frac{du}{-2}$$

$$\int_{13}^{0} \left(\frac{13-u}{2}\right)\sqrt{u}\, \frac{du}{-2}$$

$$-\frac{1}{4}\int_{13}^{0} (13-u)\sqrt{u}\, du$$

$$-\frac{1}{4}\int_{13}^{0} \left(13u^{1/2} - u^{3/2}\right) du$$

$$-\frac{1}{4}\left[\frac{13u^{3/2}}{3/2}-\frac{u^{5/2}}{5/2}\right]_{13}^{0}$$

$$-\frac{1}{4}\left[\frac{26}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right]_{13}^{0}$$

$$-\frac{1}{4}\left[(0-0)-\left(\frac{26}{3}13^{3/2}-\frac{2}{5}13^{5/2}\right)\right]$$

$$-\frac{1}{4}\left[-\frac{26}{3}13\sqrt{13}+\frac{2}{5}\times 13^2\times\sqrt{13}\right]$$

$$-\frac{1}{4}\times-\frac{26}{3}\times 13\sqrt{13}-\frac{1}{4}\times\frac{2}{5}\times 13^2\times\sqrt{13}$$

$$\frac{169}{6}\sqrt{13}-\frac{169}{10}\sqrt{13}$$

$$\boxed{\frac{169}{15}\sqrt{13}}$$