

1.

A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]



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2.

Find the term independent of x in each of the following expansions.

$$\textbf{(a)} \quad \left(3x + \frac{2}{x^2}\right)^6 \quad [3]$$

A decorative graphic at the bottom center of the page. It features a green robot arm with a circular head and a rectangular body, positioned as if it is placing or moving yellow blocks. There are several yellow blocks of different sizes and shapes scattered around the arm, including a large yellow circle and several smaller yellow squares and rectangles. The entire graphic is rendered in a simple, stylized manner with flat colors.

(b) $\left(3x + \frac{2}{x^2}\right)^6 (1 - x^3)$ [3]

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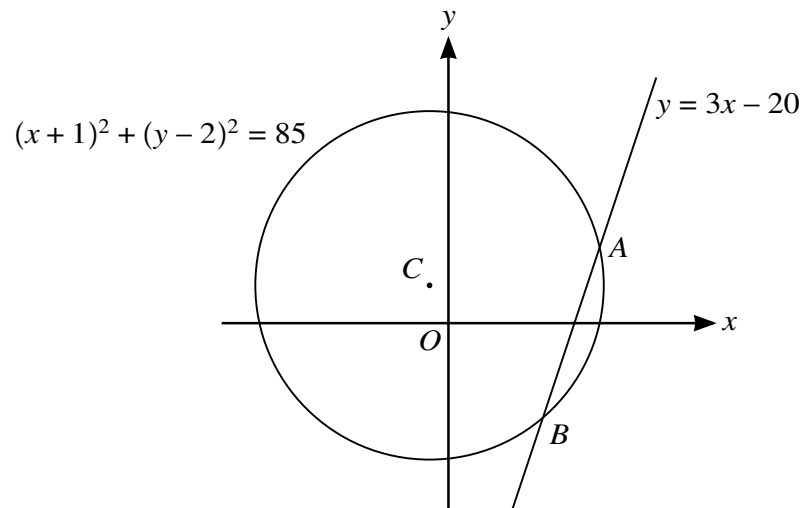
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- This image shows a single sheet of white paper with ten evenly spaced, horizontal dashed lines. The lines are black and extend across the full width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the paper.

$$\begin{aligned} f(x) &= x^2 && \text{for } x \in \mathbb{R}, \\ g(x) &= 2x^2 - 8x + 14 && \text{for } x \in \mathbb{R}. \end{aligned}$$

- # MATH TONIC

4.



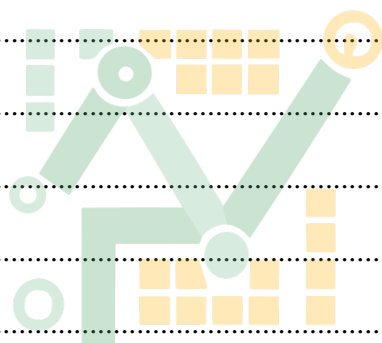
The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

- (a) Find, by calculation, the coordinates of A and B.

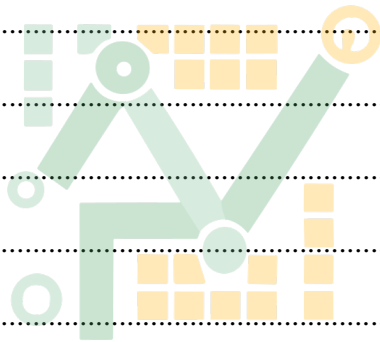
[4]

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- (b)** Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

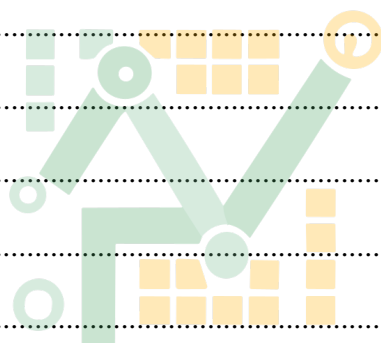


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- (b) Hence solve the equation $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} = 5$ for $0^\circ < \theta < 180^\circ$. [3]



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6.

Functions f , g and h are defined as follows:

$$f : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geq 0,$$

$g : x \mapsto mx^2 + n$ for $x \geq -2$, where m and n are constants,

$$h : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geq 0.$$

- (a)** Solve the equation $f(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a , b and c are integers. [4]



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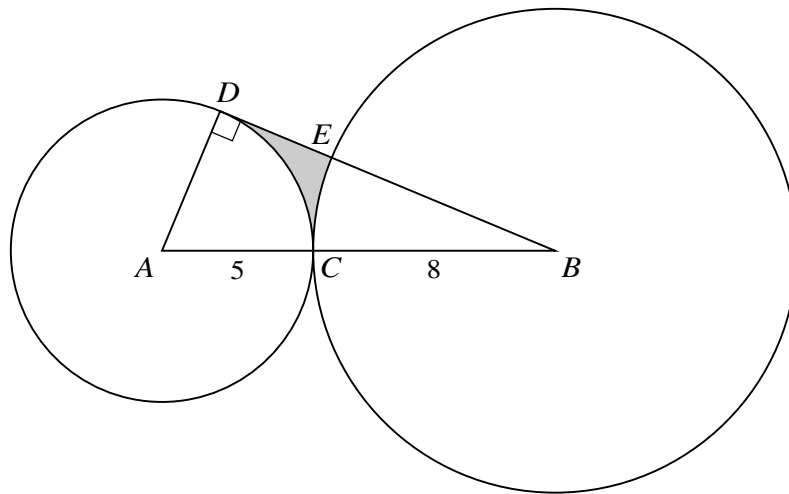
(b) Given that $f(x) \equiv gh(x)$, find the values of m and n .

[4]



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7.



The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B .

- (a) Find the perimeter of the shaded region.

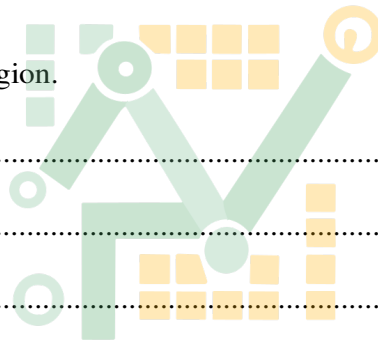
[5]



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(b) Find the area of the shaded region.

[3]



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8.

The coefficient of x^4 in the expansion of $(3+x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a .

[4]



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9.

The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

- (a) Given that the curve and the line intersect at the points with x -coordinates 0 and $\frac{3}{4}$, find the values of k and a . [4]



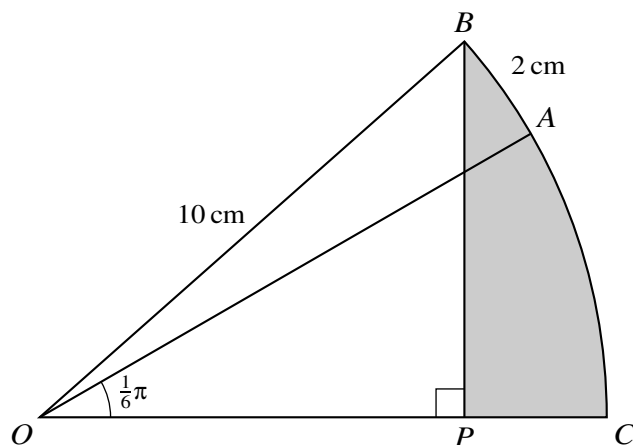
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- (b)** Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]



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10.



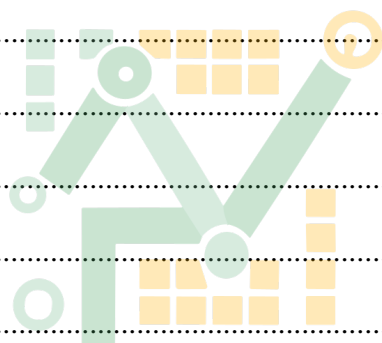
The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm . The point P lies on OC and BP is perpendicular to OC . Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm .

(a) Find the angle BOC .

[2]

Handwriting practice lines with a large watermark reading "MATH TONIC" and a stylized robot logo.

- (b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

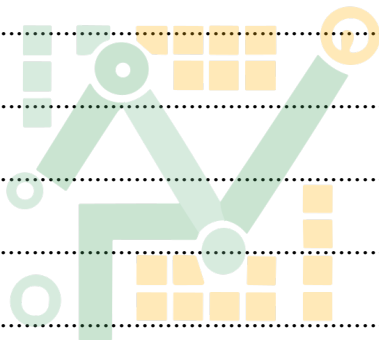


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11.

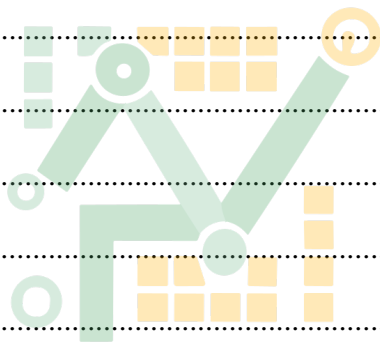
The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A(1, 1)$ and $B(2, -6)$ lie on the circle.

- (a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]



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- (b)** Find the equation of the tangent to the circle at the point A , giving your answer in the form $px + qy = k$, where p , q and k are integers. [4]



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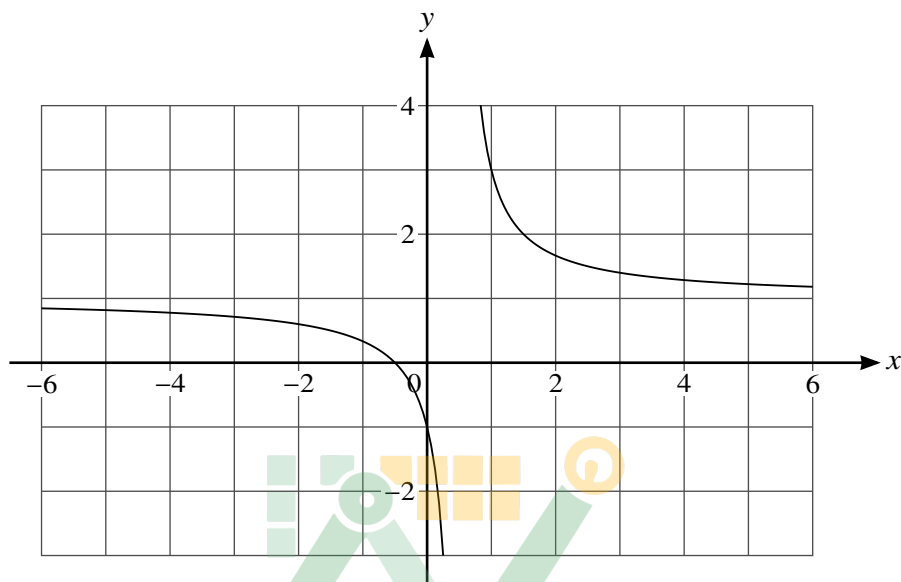
12.

Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \frac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1} .

[1]

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(b) Find an expression for $f^{-1}(x)$.

[3]

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(c) Find $gf^{-1}(3)$.

[2]

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- (d) Explain why $g^{-1}(x)$ cannot be found.

[1]

- (e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x - and y -axes. [6]

[6]



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13.

The function f is given by $f(x) = 4 \cos^4 x + \cos^2 x - k$ for $0 \leq x \leq 2\pi$, where k is a constant.

- (a)** Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$.

[5]



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- (b)** Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]



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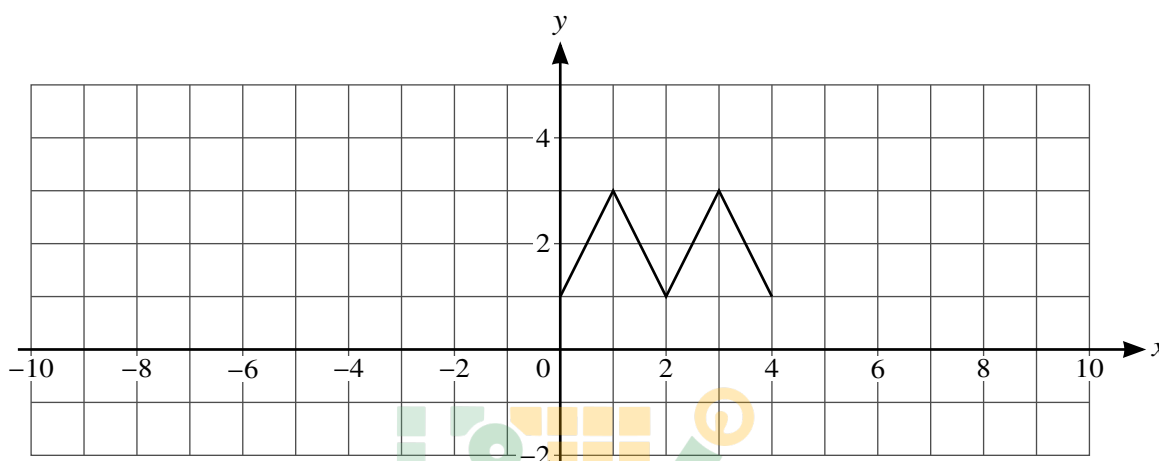
14.

The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x -direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

- (a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$.

[3]



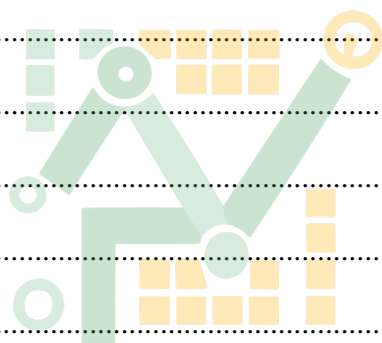
- (b)** Find an expression for $g(x)$ in terms of $f(x)$.

[2]


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15.

- (a) Prove the identity $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \equiv \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$. [3]



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Functions f and g are defined by

$$\begin{aligned} f(x) &= x + \frac{1}{x} \quad \text{for } x > 0, \\ g(x) &= ax + 1 \quad \text{for } x \in \mathbb{R}, \end{aligned}$$

where a is a constant.

- (a) Find an expression for $gf(x)$. [1]

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- (b) Given that $gf(2) = 11$, find the value of a . [2]

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- (c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

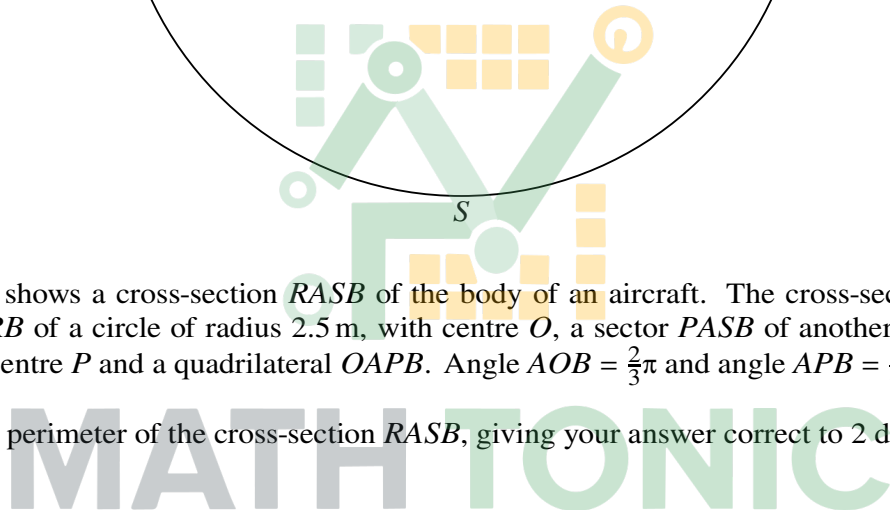
- (d)** Find and simplify an expression for $g^{-1}f(x)$. [3]

A decorative graphic at the bottom center of the page. It features a stylized house shape composed of green and yellow blocks. The house has a green roof, green walls, and a yellow chimney. There are also yellow blocks around the base of the house. A yellow circle with a green outline is positioned to the right of the house.

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- (e) Explain why the composite function fg cannot be formed. [1]

[illegible]




(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places. [3]

[illegible]

- (b) Find the difference in area of the two triangles AOB and APB , giving your answer correct to 2 decimal places. [2]



- (c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]



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