1 A bag contains 9 blue marbles and 3 red marbles. One marble is chosen at random from the bag. If this marble is blue, it is replaced back into the bag. If this marble is red, it is **not** returned to the bag. A second marble is now chosen at random from the bag.

(a) Find the probability that both the marbles chosen are red.   [1]

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(b) Find the probability that the first marble chosen is blue given that the second marble chosen is red.
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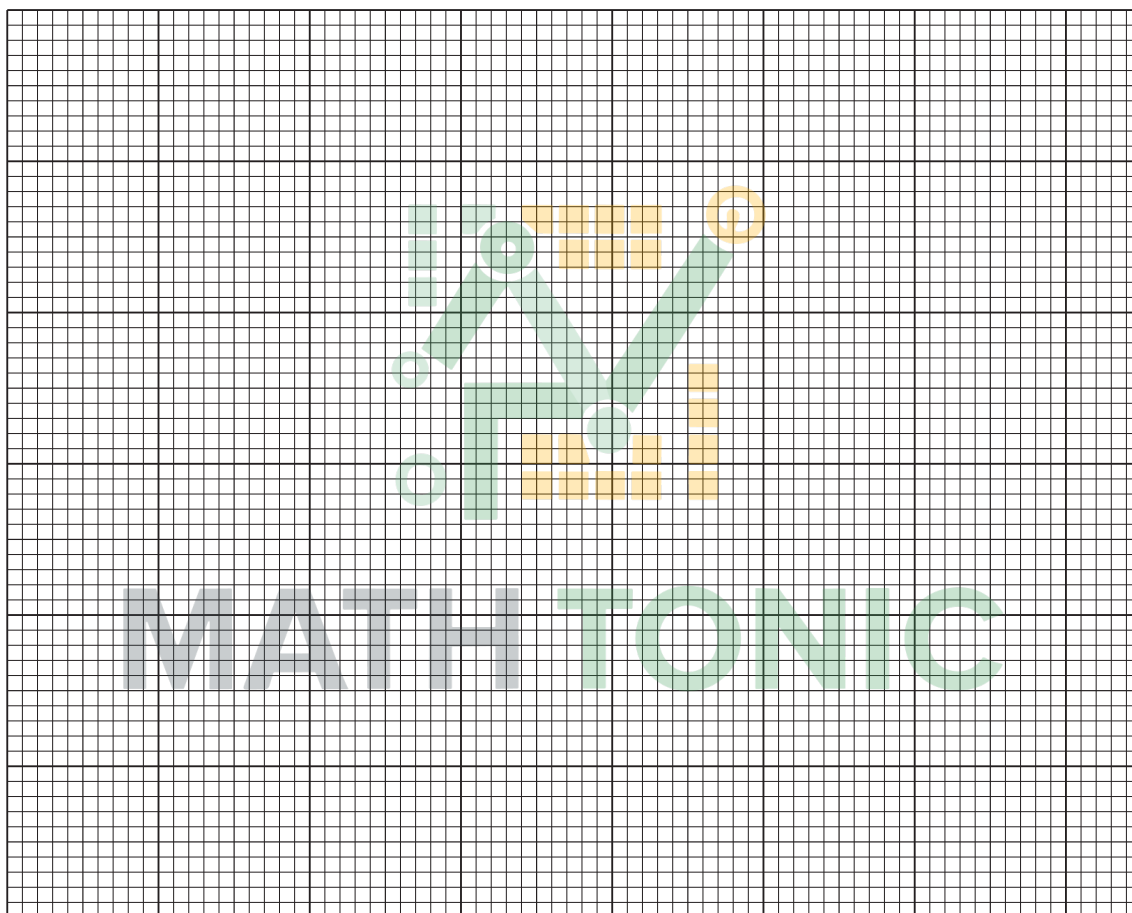
2.

The times taken, in minutes, by 150 students to complete a puzzle are summarised in the table.
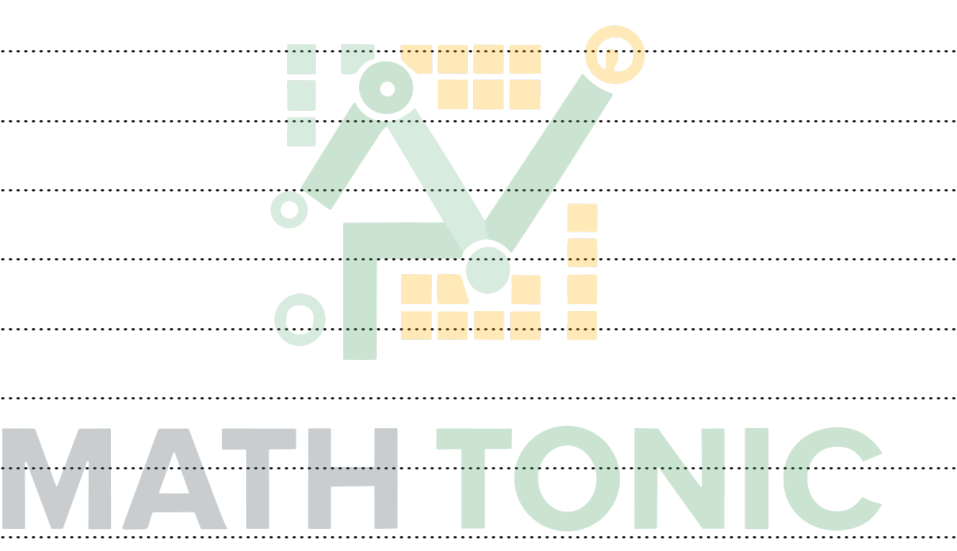
Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 70$
Frequency	8	23	35	52	20	12

(a) Draw a histogram to represent this information. [4]

(b) Calculate an estimate for the mean time for these students to complete the puzzle. [3]

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(c) In which class interval does the lower quartile of the times lie? [1]

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3.

Anil is taking part in a tournament. In each game in this tournament, players are awarded 2 points for a win, 1 point for a draw and 0 points for a loss. For each of Anil's games, the probabilities that he will win, draw or lose are 0.5, 0.3 and 0.2 respectively. The results of the games are all independent of each other.

The random variable X is the total number of points that Anil scores in his first 3 games in the tournament.

(a) Show that $P(X = 2) = 0.114$. [2]

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(b) Complete the probability distribution table for X. [3]

x	0	1	2	3	4	5	6
$P(X=x)$			0.114	0.207	0.285		0.125

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(c) Find the value of Var(X). [3]

4.

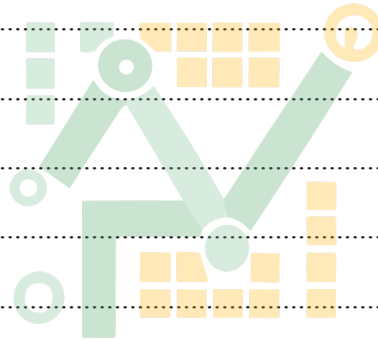A new village social club has 10 members of whom 6 are men and 4 are women. The club committee will consist of 5 members.

(a) In how many ways can the committee of 5 members be chosen if it must include at least 2 men and at least 1 woman? [4]

The 10 members of the club stand in a line for a photograph.

(b) How many different arrangements are there of the 10 members if all the men stand together and all the women stand together? [2]

For a second photograph, the members stand in two rows, with 6 on the back row and 4 on the front row. Olly and his sister Petra are two of the members of the club.

(c) How many different arrangements are there of the 10 members in which Olly and Petra stand next to each other on the front row? [4]

5.

A summary of 20 values of x gives

$$\sum(x-30) = 439, \qquad \sum(x-30)^2 = 12\,405.$$

A summary of another 25 values of x gives

$$\sum(x-30) = 470, \qquad \sum(x-30)^2 = 11346.$$

(a) Find the mean of all 45 values of x. [2]

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(b) Find the standard deviation of all 45 values of x. [2]

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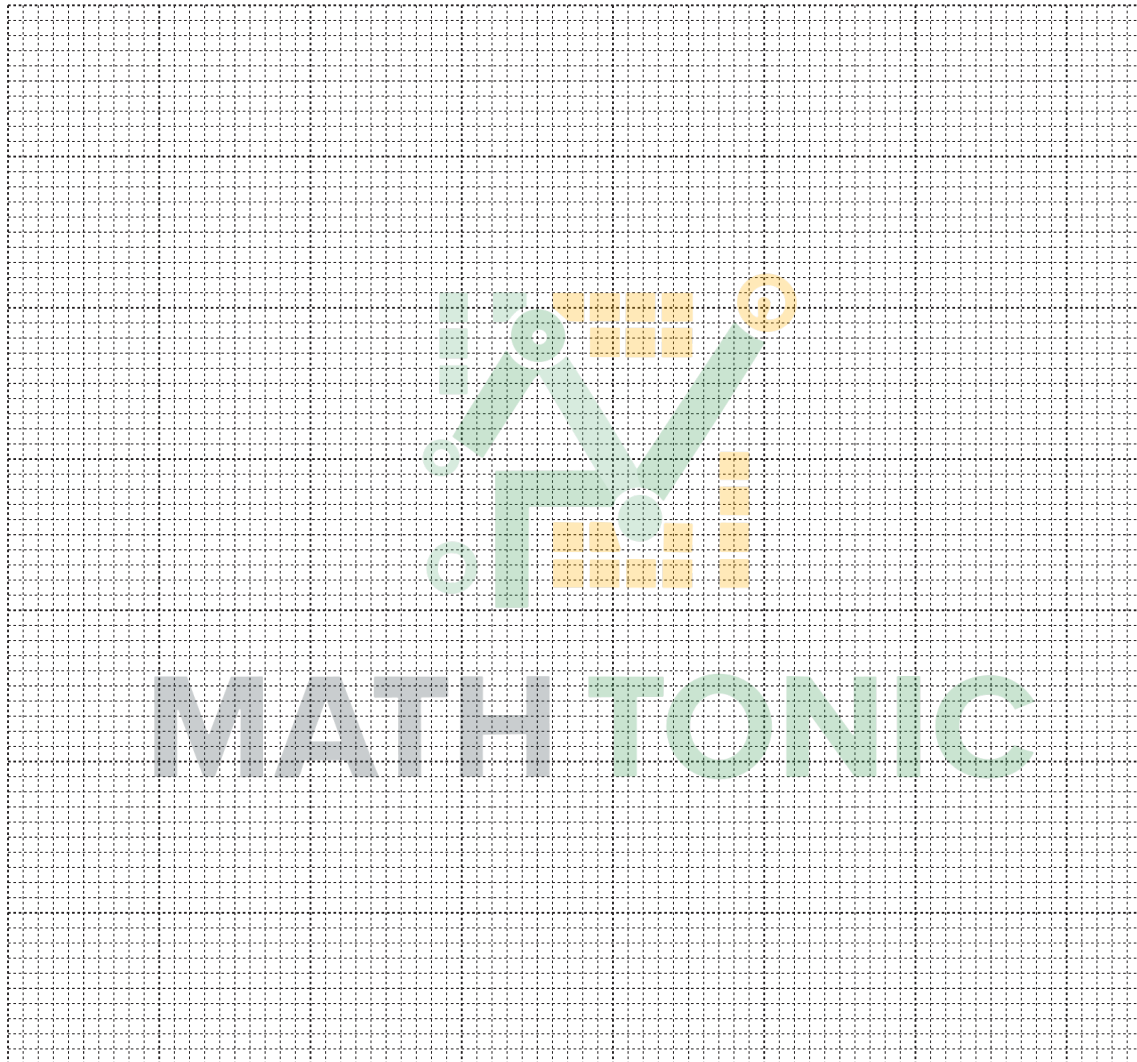
6.   The heights, in cm, of 200 adults in Barimba are summarised in the following table.

Height (h cm)	$130 \leqslant h < 150$	$150 \leqslant h < 160$	$160 \leqslant h < 170$	$170 \leqslant h < 175$	$175 \leqslant h < 195$
Frequency	16	32	76	64	12

(a) Draw a histogram to represent this information.   [4]

(b) The interquartile range is R cm. Show that R is **not** greater than 15. [2]

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7.

A game for two players is played using a fair 4-sided dice with sides numbered 1, 2, 3 and 4. One turn consists of throwing the dice repeatedly up to a maximum of three times. When a 4 is obtained, no further throws are made during that turn. A player who obtains a 4 in their turn scores 1 point.

(a) Show that the probability that a player obtains a 4 in one turn is $\frac{37}{64}$. [2]

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Xeno and Yao play this game.

(b) Find the probability that neither Xeno nor Yao score any points in their first two turns. [1]

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(c) Xeno and Yao each have three turns.

Find the probability that Xeno scores 2 more points than Yao. [3]

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8.

Harry has three coins:
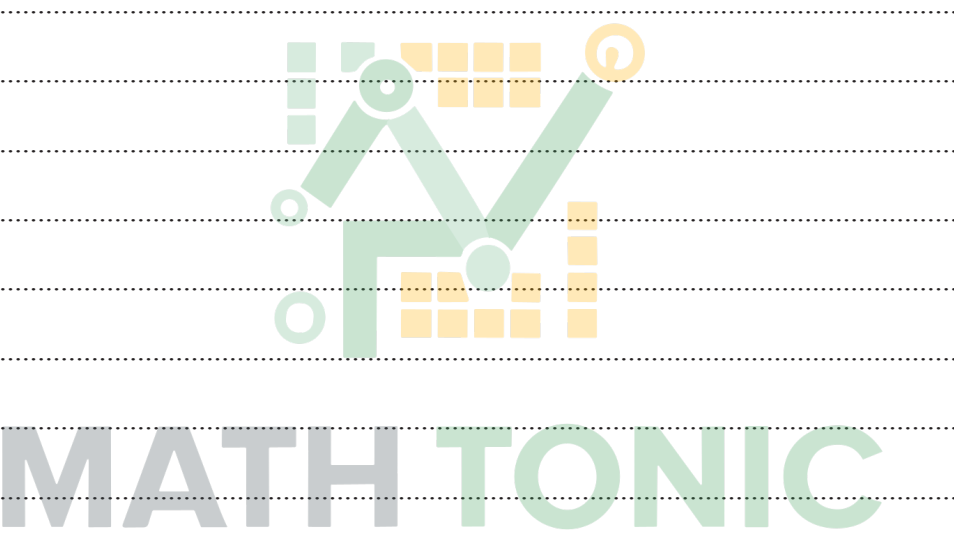
- One coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$.
- The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.
- The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$.

Harry throws the three coins. The random variable X is the number of heads that he obtains.

(a) Draw up the probability distribution table for X. [4]

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Harry has two other coins, each of which is biased so that the probability of obtaining a head when it is thrown is p. He throws all five coins at the same time. The random variable Y is the number of heads that he obtains.

(b) Given that $P(Y = 0) = 6P(Y = 5)$, find the value of p. [3]

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9.

The eight digits 1, 2, 2, 3, 4, 4, 4, 5 are arranged in a line.

(a) How many different arrangements are there of these 8 digits? [1]

(b) Find the number of different arrangements of the 8 digits in which there is a 2 at the beginning, a 2 at the end and the three 4s are not all together. [4]

Three digits are selected at random from the eight digits 1, 2, 2, 3, 4, 4, 4, 5.

(c) Find the probability that the three digits are all different. [5]

Seva has a coin which is biased so that when it is thrown the probability of obtaining a head is $\frac{1}{3}$. He also has a bag containing 4 red marbles and 5 blue marbles.
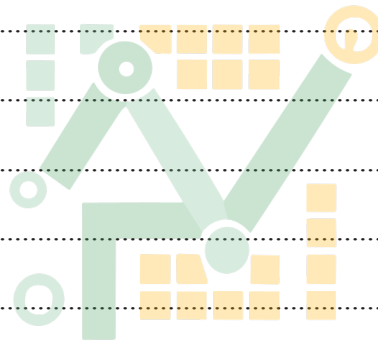
Seva throws the coin. If he obtains a head, he selects one marble from the bag at random. If he obtains a tail, he selects two marbles from the bag at random and without replacement.

(a) Find the probability that Seva selects at least one red marble. [3]

(b) Find the probability that Seva obtains a head given that he selects no red marbles. [2]

11.

The back-to-back stem-and-leaf diagram shows the annual salaries of 19 employees at each of two companies, Petral and Ravon.

Petral							Ravon				
			3	0	0	30	2	6			
9	9	8	2	2	1	31	1	5			
		5	5	4	0	32	0	0	2		
			7	5	3	33	0	4	8	9	
				1	0	34	1	1	3	4	6
						35	3				
					8	36	7	9			

Key: 2 | 31 | 5 means \$31 200 for a Petral employee and \$31 500 for a Ravon employee.

(a) Find the median and the interquartile range of the salaries of the Petral employees.   [3]

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The median salary of the Ravon employees is \$33 800, the lower quartile is \$32 000 and the upper quartile is \$34 400.

(b) Represent the data shown in the back-to-back stem-and-leaf diagram by a pair of box-and-whisker plots in a single diagram. [3]
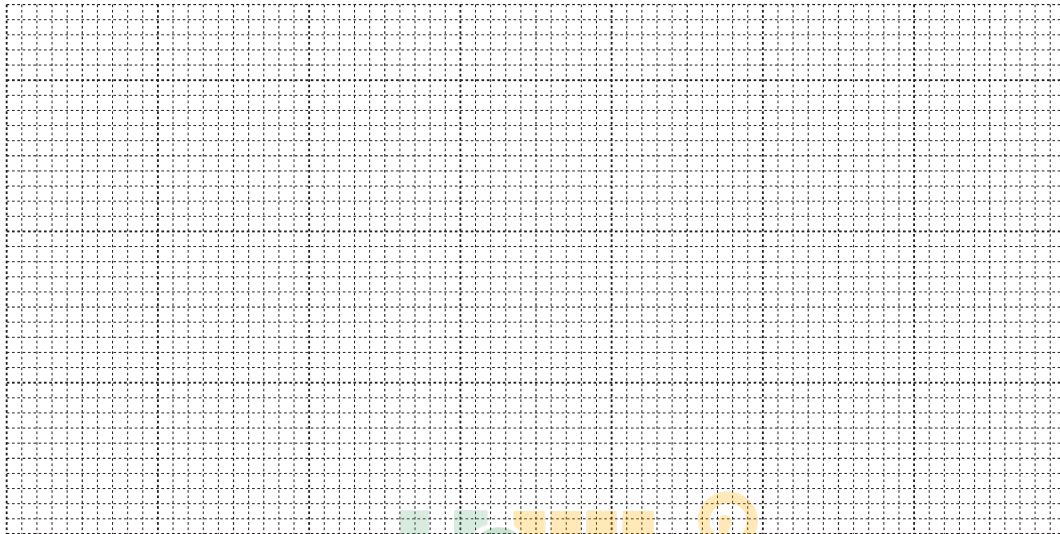
(c) Comment on whether the mean or the median would be a better representation of the data for the employees at Petral. [1]
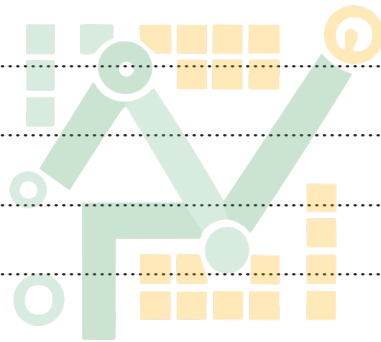
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12.

Jasmine has one $5 coin, two $2 coins and two $1 coins. She selects two of these coins at random. The random variable X is the total value, in dollars, of these two coins.

(a) Show that $P(X = 7) = 0.2$. [1]

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(b) Draw up the probability distribution table for X. [3]

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(c) Find the value of $\text{Var}(X)$. [3]

(a) How many different arrangements are there of the 10 letters in the word REGENERATE?   [1]

(b) How many different arrangements are there of the 10 letters in the word REGENERATE in which the 4 Es are together and the 2 Rs have exactly 3 letters in between them?   [4]

(c) Find the probability that a randomly chosen arrangement of the 10 letters in the word REGENERATE is one in which the consonants (G, N, R, R, T) and vowels (A, E, E, E, E) alternate, so that no two consonants are next to each other and no two vowels are next to each other. [5]

14.

The numbers on the faces of a fair six-sided dice are 1, 2, 2, 3, 3, 3. The random variable X is the total score when the dice is rolled twice.

(a) Draw up the probability distribution table for X. [3]

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(b) Find the value of Var(X). [3]

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(c) Find the probability that X is even given that $X > 3$. [2]

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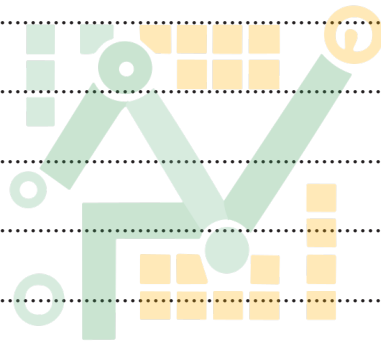
15.

Box *A* contains 6 green balls and 3 yellow balls.

Box *B* contains 4 green balls and *x* yellow balls.

A ball is chosen at random from box *A* and placed in box *B*. A ball is then chosen at random from box *B*.

(a) Draw a tree diagram to represent this information, showing the probability on each of the branches.

[4]
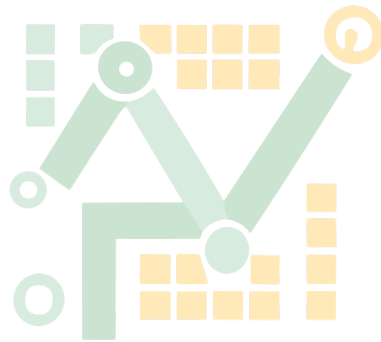
The probability that both the balls chosen are the same colour is $\frac{8}{15}$.

(b) Find the value of x. [3]

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16.

The times taken, in seconds, by 15 members of each of two swimming clubs, the Penguins and the Dolphins, to swim 50 metres are shown in the following table.

Penguins	35	39	42	44	45	45	48	50	56	58	59	61	66	68	72
Dolphins	36	41	43	48	49	49	50	51	54	56	56	60	61	64	71

(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Penguins on the left-hand side. [4]
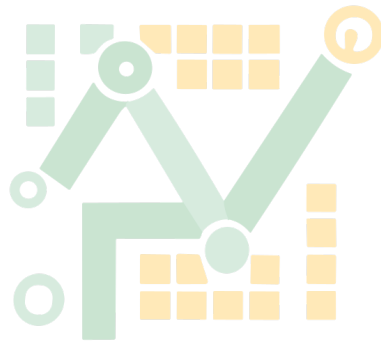
The diagram shows a box-and-whisker plot representing the times for the Penguins.

(b) On the same diagram, draw a box-and-whisker plot to represent the times for the Dolphins. [3]
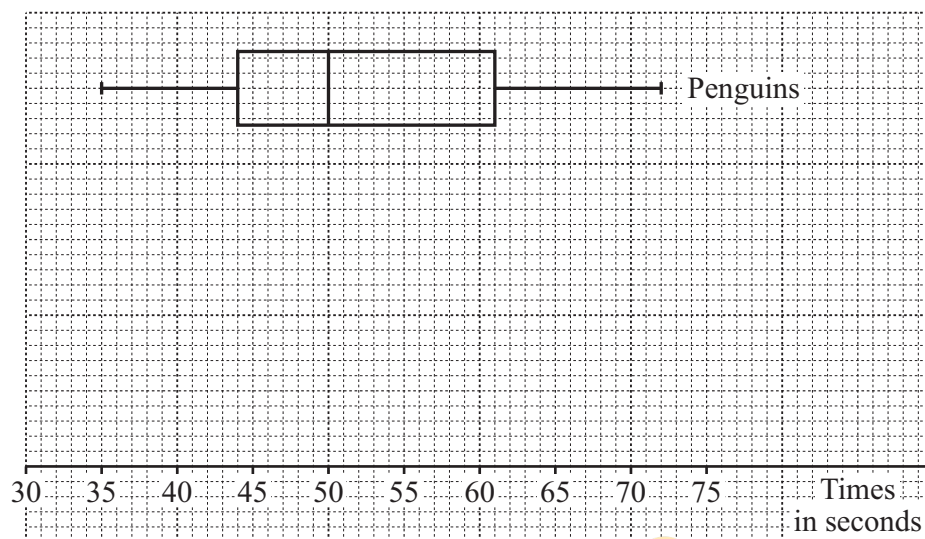
(c) Hence state **one** difference between the distributions of the times for the Penguins and the Dolphins. [1]

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17.

(a) How many different arrangements are there of the 9 letters in the word RECORDERS? [1]

(b) How many different arrangements are there of the 9 letters in the word RECORDERS in which there is an E at the beginning, an E at the end and the three Rs are not all together? [3]

The 9 letters of the word RECORDERS are divided at random into two groups: a group of 5 letters and a group of 4 letters.

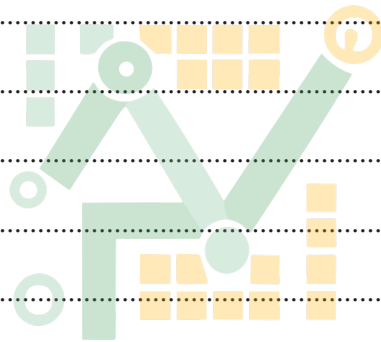**(c)** Find the probability that the three Rs are in the same group. [4]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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