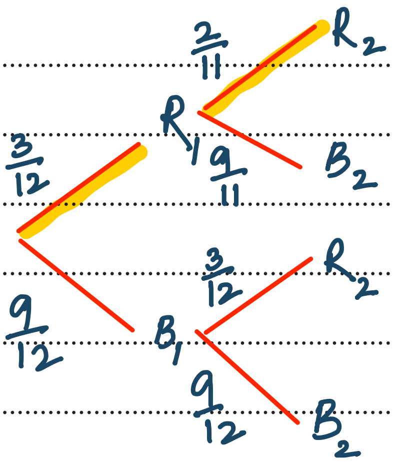
1 A bag contains 9 blue marbles and 3 red marbles. One marble is chosen at random from the bag. If this marble is blue, it is replaced back into the bag. If this marble is red, it is **not** returned to the bag. A second marble is now chosen at random from the bag.

(a) Find the probability that both the marbles chosen are red. [1]



$$P(R \cap R) = \frac{3}{12} \times \frac{2}{11}$$

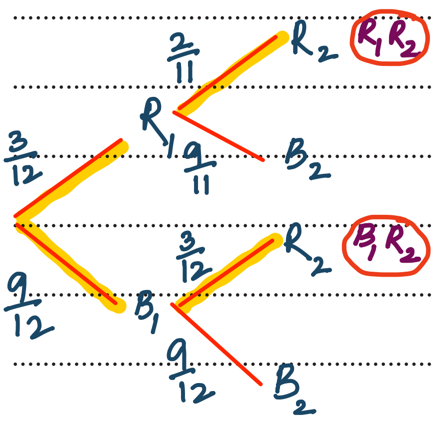
$$\boxed{P(R \cap R) = \frac{1}{22}}$$

(b) Find the probability that the first marble chosen is blue given that the second marble chosen is red. [3]

conditional probability

P (First marble chosen is Blue / Second marble Red)

$$P(B_1 / R_2) = \frac{P(B_1 \cap R_2)}{P(R_2)}$$



$$= \frac{P(B_1 \cap R_2)}{P(R_1 \cap R_2) + P(B_1 \cap R_2)}$$

$$= \frac{\frac{9}{12} \times \frac{3}{12}}{\frac{3}{12} \times \frac{2}{11} + \frac{9}{12} \times \frac{3}{12}}$$

$$= \frac{\frac{3}{16}}{\frac{41}{176}}$$

$$= \frac{33}{41}$$

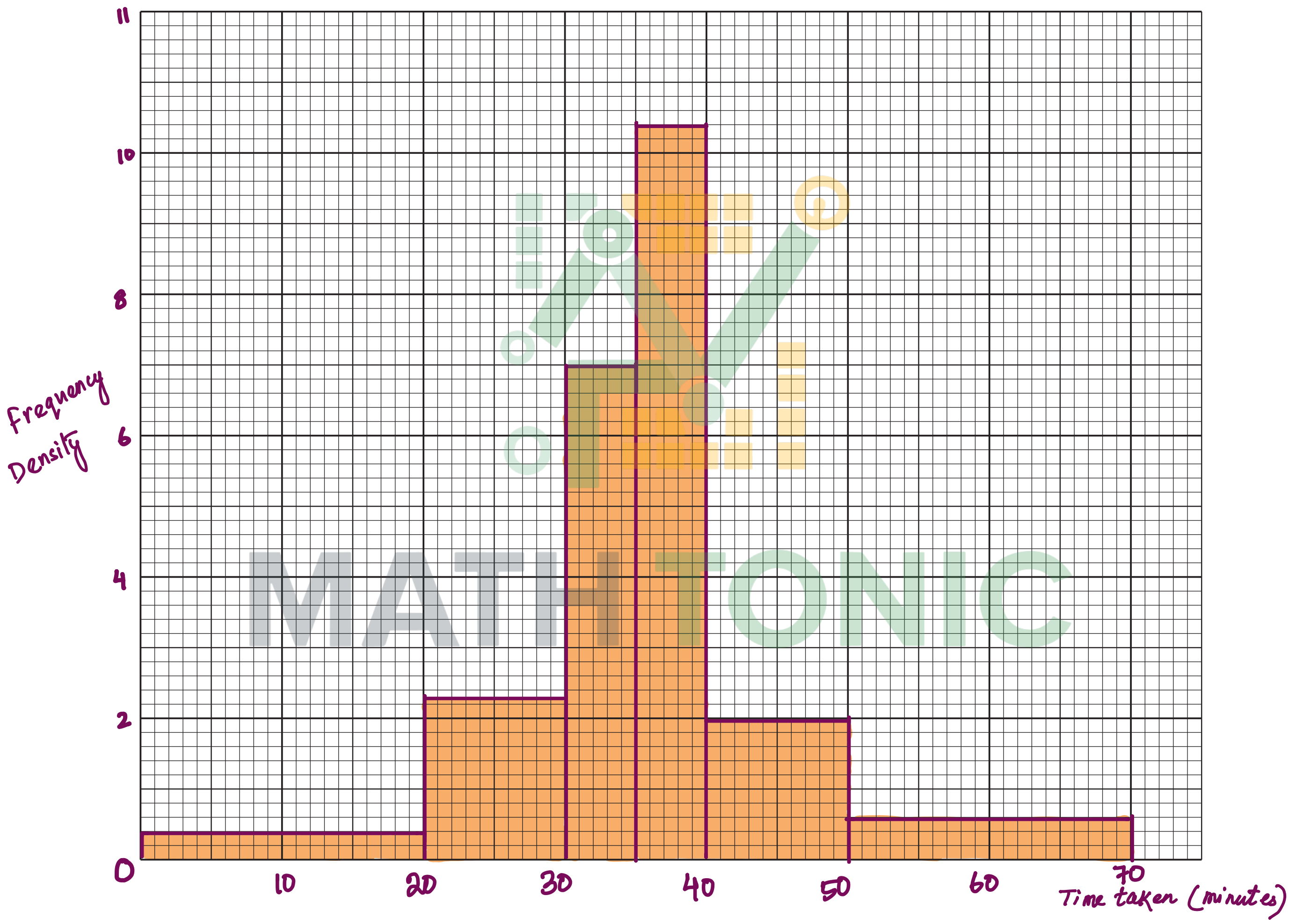
2.

The times taken, in minutes, by 150 students to complete a puzzle are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 70$
Frequency	8	23	35	52	20	12
Class width	20	10	5	5	10	20
Frequency Density	$\frac{8}{20} = 0.4$	$\frac{23}{10} = 2.3$	$\frac{35}{5} = 7$	$\frac{52}{5} = 10.4$	$\frac{20}{10} = 2$	$\frac{12}{20} = 0.6$

$$\boxed{\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class width}}}$$

(a) Draw a histogram to represent this information. [4]



(b) Calculate an estimate for the mean time for these students to complete the puzzle. [3]

Time Taken	Midpoint (t)	Frequency (f)	$f \times t$
$0 \leq t < 20$	10	8	80
$20 \leq t < 30$	25	23	575
$30 \leq t < 35$	32.5	35	1137.5
$35 \leq t < 40$	37.5	52	1950
$40 \leq t < 50$	45	20	900
$50 \leq t < 70$	60	12	720

$\Sigma f = 150$ $\Sigma ft = 5362.5$

$$\text{Mean time } (\bar{t}) = \frac{5362.5}{150}$$

$$= 35.75$$

(c) In which class interval does the lower quartile of the times lie? [1]

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 70$
Frequency	8	23	35	52	20	12
Cumulative frequency	8	31	66	118	138	150

37.5^{th} value lies here

$Q_1 = \frac{1}{4} \times 150 = 37.5^{th}$ value

So, $\boxed{Q_1 \text{ lies in } 30 \leq t < 35}$

3.

Anil is taking part in a tournament. In each game in this tournament, players are awarded 2 points for a win, 1 point for a draw and 0 points for a loss. For each of Anil's games, the probabilities that he will win, draw or lose are 0.5, 0.3 and 0.2 respectively. The results of the games are all independent of each other.

The random variable X is the total number of points that Anil scores in his first 3 games in the tournament.

(a) Show that $P(X = 2) = 0.114$. [2]

followings are the possibilities for total score of 2 in three games.

2 W	0 L	0 L	$0.5 \times 0.2 \times 0.2 = 0.02$	
0 L	2 W	0 L	$0.2 \times 0.5 \times 0.2 = 0.02$	0.02×3
0 L	0 L	2 W	$0.2 \times 0.2 \times 0.5 = 0.02$	
1 D	1 D	0 L	$0.3 \times 0.3 \times 0.2 = 0.018$	
1 D	0 L	1 D	$0.3 \times 0.2 \times 0.3 = 0.018$	0.018×3
0 L	1 D	1 D	$0.2 \times 0.3 \times 0.3 = 0.018$	

$$P(X=2) = (0.02 \times 3) + (0.018 \times 3)$$
$$= 0.114$$

(b) Complete the probability distribution table for X. [3]

x	0	1	2	3	4	5	6
$P(X = x)$	0.008	0.036	0.114	0.207	0.285	0.225	0.125

$$P(X=0) = P(L \cap L \cap L) = 0.2 \times 0.2 \times 0.2 = 0.008$$

$$P(X=1) = P(D \cap L \cap L) + P(L \cap D \cap L) + P(L \cap L \cap D)$$
$$= (0.3 \times 0.2 \times 0.2) + (0.2 \times 0.3 \times 0.2) + (0.2 \times 0.2 \times 0.3)$$
$$= 0.036$$

$P(X=5) = 1-(0.008+0.036+0.114+0.207+0.285+0.125)$

$= 0.225$

(c) Find the value of Var(X). [3]

x	0	1	2	3	4	5	6	
$P(X=x)$	0.008	0.036	0.114	0.207	0.285	0.225	0.125	
$x \cdot P$	0	0.036	0.228	0.621	1.140	1.125	0.750	$\Sigma x \cdot p = 3.9$
$x^2 \cdot P$	0	0.036	0.456	1.863	4.560	5.625	4.5	$\Sigma x^2 p = 17.04$

$E(X) = \Sigma x p$
$E(X^2) = \Sigma x^2 p$

$Var(X) = E(X^2) - [E(X)]^2$

$= 17.04 - (3.9)^2$

$= 1.83$

4. A new village social club has 10 members of whom 6 are men and 4 are women. The club committee will consist of 5 members.

(a) In how many ways can the committee of 5 members be chosen if it must include at least 2 men and at least 1 woman? [4]

possibilities: Committee of 5 members

MEN (6)	WOMEN (4)	
2	3	${}^6C_2 \times {}^4C_3 = 60$
3	2	${}^6C_3 \times {}^4C_2 = 120$
4	1	${}^6C_4 \times {}^4C_1 = 60$

Total no. of ways $= 60 + 120 + 60$

$= 240$

The 10 members of the club stand in a line for a photograph.

(b) How many different arrangements are there of the 10 members if all the men stand together and all the women stand together? [2]

MEN: _ _ _ _ _ _ → $6!$

WOMEN: _ _ _ _ → $4!$

No. of arrangements: $6! \times 4! \times 2$ ← way of swapping men and women group

$= 34560$

For a second photograph, the members stand in two rows, with 6 on the back row and 4 on the front row. Olly and his sister Petra are two of the members of the club.

(c) How many different arrangements are there of the 10 members in which Olly and Petra stand next to each other on the front row? [4]

These two persons are fixed. 8 persons left

Front Row: $\boxed{O \quad P}$ _ _ $2! \times {}^8P_2 \times 3$

Internal arrangement between Olly and petra.

Besides Olly and petra 8 persons are left

Out of which 2 persons are be filled.

(This Can be written as 8×7)

for OP

OP _ _

_ OP _

_ _ OP

Back Row: _ _ _ _ _ _ $6!$

Total No. of Arrangement:

$2! \times {}^8P_2 \times 3 \times 6!$

241920

5.

$$\text{Mean} = \frac{\Sigma x + \Sigma y}{n_1 + n_2}$$

A summary of 20 values of x gives

$$\Sigma(x-30) = 439, \qquad \Sigma(x-30)^2 = 12\,405.$$

A summary of another 25 values of x gives

$$\Sigma(x-30) = 470, \qquad \Sigma(x-30)^2 = 11\,346.$$

(a) Find the mean of all 45 values of x. [2]

For Summary of 20 values : $n = 20$

$$\Sigma(x-30) = 439$$

$$\Sigma x - 30n = 439$$

$$\Sigma x = 439 + 30n = 439 + 30(20) = 1039$$

For Summary of 25 values : $n = 25$

$$\Sigma(x-30) = 470$$

$$\Sigma x - 30n = 470$$

$$\Sigma x = 470 + 30n = 470 + 30(25) = 1220$$

Mean of all 45 values : $\dfrac{1039 + 1220}{45} = \boxed{50.2}$

(b) Find the standard deviation of all 45 values of x. [2]

Standard deviation of $x - 30$ = Standard deviation of x

(Standard deviation is not affected by Coding)

$$\sigma = \sqrt{\frac{\Sigma(x-30)^2 + \Sigma(y-30)^2}{n_1 + n_2} - \left[\frac{\Sigma(x-30) + \Sigma(y-30)}{n_1 + n_2}\right]^2}$$

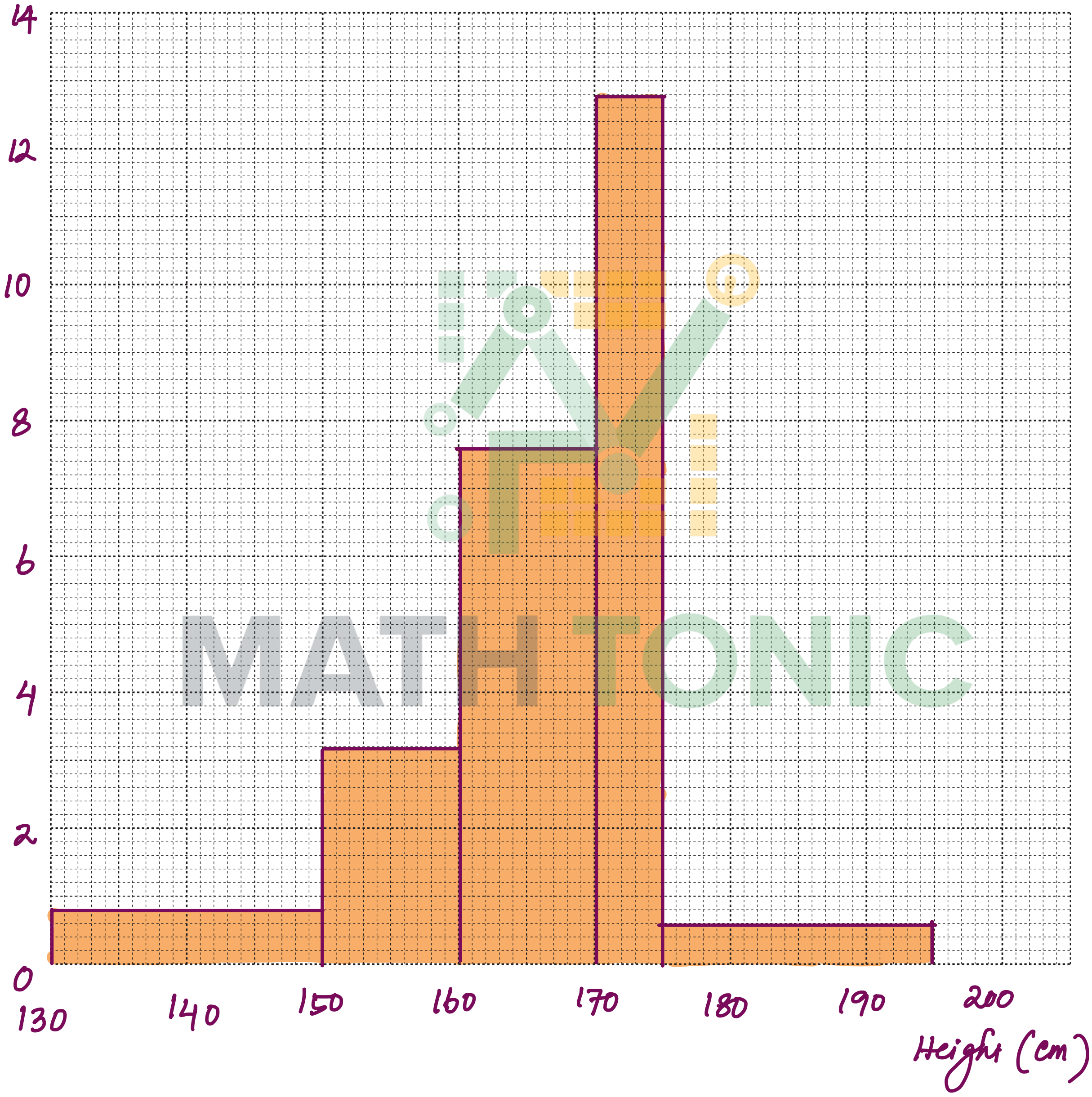
$$\sigma = \sqrt{\frac{12045 + 11346}{45} - \left(\frac{439 + 470}{45}\right)^2}$$

$$\sigma = 10.9$$

6. The heights, in cm, of 200 adults in Barimba are summarised in the following table.

Height (h cm)	$130 \leqslant h < 150$	$150 \leqslant h < 160$	$160 \leqslant h < 170$	$170 \leqslant h < 175$	$175 \leqslant h < 195$
Frequency	16	32	76	64	12
Class width	20	10	10	5	20
Frequency Density	$\frac{16}{20} = 0.8$	$\frac{32}{10} = 3.2$	$\frac{76}{10} = 7.6$	$\frac{64}{5} = 12.8$	$\frac{12}{20} = 0.6$

(a) Draw a histogram to represent this information. [4]



(b) The interquartile range is R cm. Show that R is **not** greater than 15. [2]

$n = 200$

$Q_1 = \frac{1}{4} \times 200 = 50^{th}$ person

$Q_3 = \frac{3}{4} \times 200 = 150^{th}$ person

Height (h cm)	$130 \leqslant h < 150$	$150 \leqslant h < 160$	$160 \leqslant h < 170$	$170 \leqslant h < 175$	$175 \leqslant h < 195$
Frequency	16	32	76	64	12
Cumulative frequency	16	48	124	188	200

Q_1 lies here Q_3 lies here

min

Q_1 lies in $160 \leq h < 170$

Q_3 lies in $170 \leq h < 175$

max

Inter Quartile Range (IQR) $= Q_3 - Q_1$

$IQR_{max} = Q_3(max) - Q_1(min)$

$= 175 - 160$

$R = 15$

So, R cannot be greater than 15.

7. * Geometric Distribution

A game for two players is played using a fair 4-sided dice with sides numbered 1, 2, 3 and 4. One turn consists of throwing the dice repeatedly up to a maximum of three times. When a 4 is obtained, no further throws are made during that turn. A player who obtains a 4 in their turn scores 1 point.

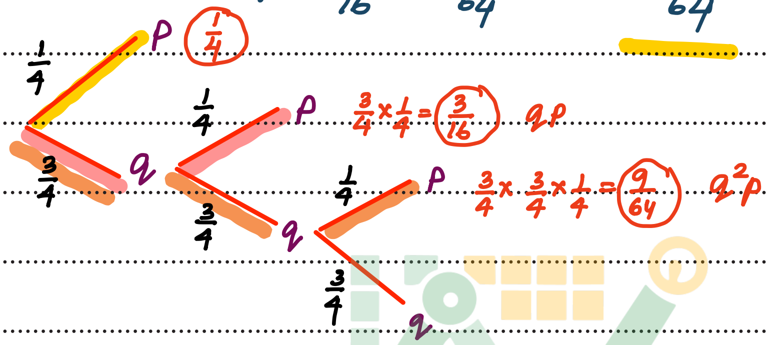
(a) Show that the probability that a player obtains a 4 in one turn is $\frac{37}{64}$. [2]

(Success) probability of obtaining four $(p) = \frac{1}{4}$

(Failure) Not obtaining four $(q) = \frac{3}{4}$

$P(X=1) = p + qp + q^2p$

$= \frac{1}{4} + \frac{3}{16} + \frac{9}{64} = \frac{37}{64}$



Alternative Method:

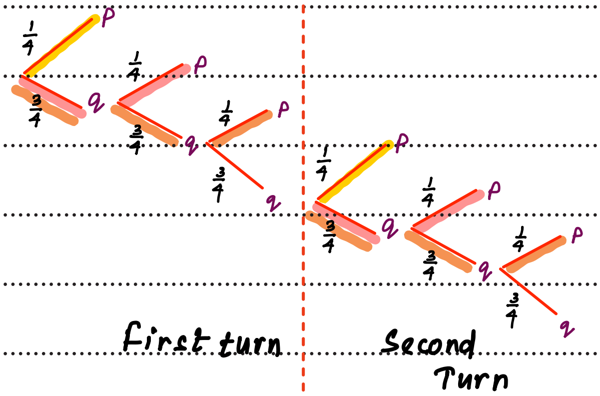
$1 - q^3 = 1 - \left(\frac{3}{4}\right)^3 = \frac{37}{64}$

Xeno and Yao play this game.

(b) Find the probability that neither Xeno nor Yao score any points in their first two turns. [1]

XENO

Same for YAO as well :



(Failure) Not obtaining four $(q) = \frac{3}{4}$

$P(\text{Neither xeno nor Yano}) = P(X) \times P(Y) = \left(\frac{3}{4}\right)^6 \times \left(\frac{3}{4}\right)^6$

$= 0.0317$

(c) Xeno and Yao each have three turns.

Find the probability that Xeno scores 2 more points than Yao. [3]

	Xeno	Yao
Possibilities :	2	0
	3	1

$$W \sim B\left(3, \frac{37}{64}\right)$$

Xeno(2), yao(0)

$$P(X=2) \times P(Y=0)$$

$$^3C_2\left(\frac{37}{64}\right)^2 \times \left(\frac{27}{64}\right) \times {}^3C_0\left(\frac{37}{64}\right)^0 \times \left(\frac{27}{64}\right)^3$$

$$= 0.03176$$

Xeno(3), Yao(1) :

$$P(X=3) \times P(Y=1)$$

$$^3C_3 \times \left(\frac{37}{64}\right)^3 \times \left(\frac{27}{64}\right)^0 \times {}^3C_1 \times \left(\frac{37}{64}\right)^1 \times \left(\frac{27}{64}\right)^2$$

$$= 0.05964$$

$$0.03176 + 0.05964 = \underline{0.0914}$$

8.

Harry has three coins:

- One coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$.
- The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.
- The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$.

Harry throws the three coins. The random variable X is the number of heads that he obtains.

(a) Draw up the probability distribution table for X. [4]

	1^{st} coin	2^{nd} coin	3^{rd} coin	
1H	H	T	T	$\frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$
	T	H	T	$\frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{2}{15}$
	T	T	H	$\frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1}{10}$
2H	H	H	T	$\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{1}{15}$
	H	T	H	$\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1}{20}$
	T	H	H	$\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{30}$
3H	H	H	H	$\frac{1}{3} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$
0H	T	T	T	$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

For 1 Head

$\frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{13}{30}$

For 2 Heads

$\frac{1}{15} + \frac{1}{20} + \frac{1}{30} = \frac{3}{20}$

For 3 Heads : $\frac{1}{60}$

For 0 Head: $\frac{2}{5}$

Probability distribution table :

x	0	1	2	3
$P(X=x)$	$2/5$	$13/30$	$3/20$	$1/60$

Harry has two other coins, each of which is biased so that the probability of obtaining a head when it is thrown is p. He throws all five coins at the same time. The random variable Y is the number of heads that he obtains.

(b) Given that $P(Y = 0) = 6P(Y = 5)$, find the value of p. [3]

$P(H) = p \qquad P(T) = 1-p$

$P(Y=0) = P(T \cap T \cap T \cap T \cap T)$

No Head

1st 2nd 3rd 4th 5th

$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)(1-p)$

$P(Y=0) = \frac{2}{5}(1-p)^2$

$P(Y=5) = P(H \cap H \cap H \cap H \cap H)$

No Tail

$= \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p \times p$

$P(Y=5) = \frac{p^2}{60}$

Given that:

$P(Y=0) = 6P(Y=5)$

$\frac{2}{5}(1-p)^2 = 6\frac{p^2}{60}$

$60 \times 2(1-p)^2 = 5 \times 6p^2$

$120(1-p)^2 = 30p^2$

$4(1-p)^2 = p^2$

$4(1-2p+p^2) = p^2$

$4 - 8p + 4p^2 = p^2$

$4p^2 - p^2 - 8p + 4 = 0$

$3p^2 - 8p + 4 = 0$

$(3p-2)(p-2) = 0$

$p = \frac{2}{3}$ or $p = 2$

probability can't be greater than 1.

$\boxed{p = \frac{2}{3}}$

9. www.mathtonic.com

The eight digits 1, 2, 2, 3, 4, 4, 4, 5 are arranged in a line.

(a) How many different arrangements are there of these 8 digits? [1]

2 = 2 Nos

4 = 3 Nos

$$\frac{8!}{2! \times 3!} = 3360$$

2! → 2 two's

3! → 3 four's

(b) Find the number of different arrangements of the 8 digits in which there is a 2 at the beginning, a 2 at the end and the three 4s are not all together. [4]

2 at the beginning & 2 at the end: 2 _ _ _ _ _ _ 2

$$\frac{6!}{3!}$$

3! → 3 fours

2 at the beginning and end & all three fours together: 2 $\boxed{4\ 4\ 4}$ _ _ _ 2

4!

So, 2's at each end and 3 fours are not all together:

$$\frac{6!}{3!} - 4!$$

$$120 - 24 = 96$$

Three digits are selected at random from the eight digits 1, 2, 2, 3, 4, 4, 4, 5.

(c) Find the probability that the three digits are all different. [5]

possibilities

(2, 2) Digit 2	(4, 4, 4) Digit 4	(1, 3, 5) Other Digits	
0	0	3	${}^2C_0 \times {}^3C_0 \times {}^3C_3 = 1$
0	1	2	${}^2C_0 \times {}^3C_1 \times {}^3C_2 = 9$
1	0	2	${}^2C_1 \times {}^3C_0 \times {}^3C_2 = 6$
1	1	1	${}^2C_1 \times {}^3C_1 \times {}^3C_1 = 18$

Total = 34

Without any restriction:

$${}^8C_3 = 56$$

Probability: $\frac{34}{56}$

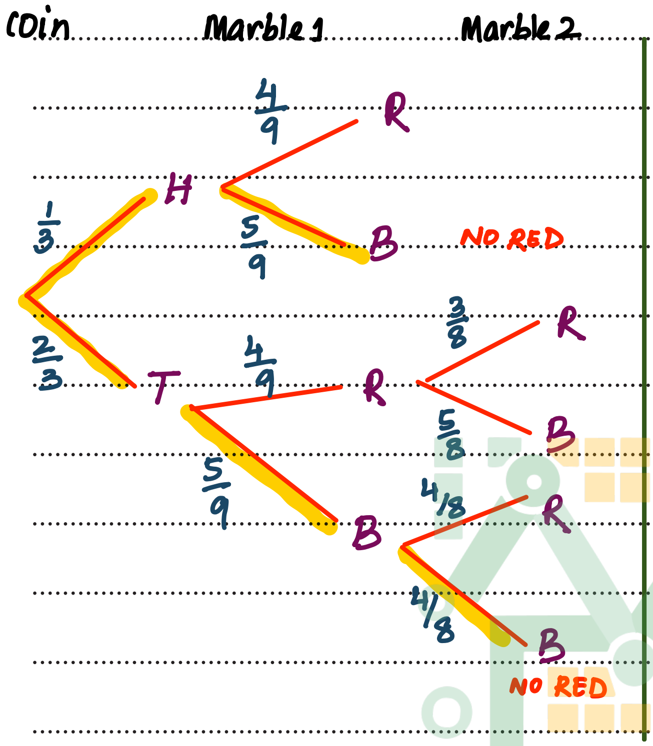
$$= \frac{17}{28}$$

10.

Seva has a coin which is biased so that when it is thrown the probability of obtaining a head is $\frac{1}{3}$. He also has a bag containing 4 red marbles and 5 blue marbles.

Seva throws the coin. If he obtains a head, he selects one marble from the bag at random. If he obtains a tail, he selects two marbles from the bag at random and without replacement.

(a) Find the probability that Seva selects at least one red marble. [3]



P(At least One Red)

$= 1 - P(\text{No red})$

$= 1 - \left[\frac{1}{3}\times\frac{5}{9} - \frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}\right]$

$= \frac{17}{27}$

(b) Find the probability that Seva obtains a head given that he selects no red marbles. [2]

conditional probability

$P(\text{Head} \mid \text{No Red}) = \dfrac{P(\text{Head} \cap \text{No Red})}{P(\text{No red})}$

$= \dfrac{\frac{1}{3}\times\frac{5}{9}}{\frac{10}{27}} = \frac{5}{27}\times\frac{27}{10}$

$= \frac{1}{2}$

If $P(\text{At Least One Red}) = \frac{17}{27}$

Then $P(\text{No red}) = 1 - \frac{17}{27} = \frac{10}{27}$

11.

The back-to-back stem-and-leaf diagram shows the annual salaries of 19 employees at each of two companies, Petral and Ravon.

Petral		Ravon
3 0 0	30	2 6
9 9 8 2 2 1	31	1 5
5 5 4 0	32	0 0 2
7 5 3	33	0 4 8 9
1 0	34	1 1 3 4 6
	35	3
8	36	7 9

Key: 2 | 31 | 5 means \$31 200 for a Petral employee and \$31 500 for a Ravon employee.

(a) Find the median and the interquartile range of the salaries of the Petral employees. [3]

$n = 19$ (odd number)

for odd number

So Median $= \left(\frac{n+1}{2}\right)^{th} = \frac{19+1}{2} = 10^{th}$ value

10^{th} value $= \$\,32000$

$Q_1 = \frac{1}{4} \times (19+1)^{th} = 5^{th}$ value $= \$\,31200$

$Q_3 = \frac{3}{4}(19+1)^{th} = 15^{th}$ value $= \$\,33500$

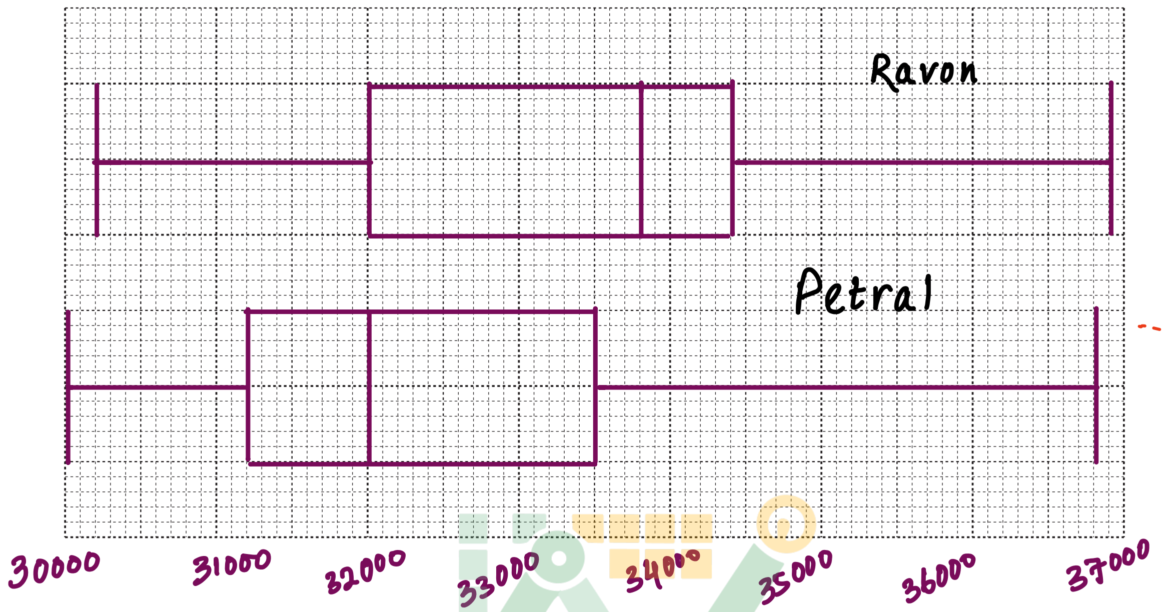
Interquartile Range $= Q_3 - Q_1$

$= 33500 - 312000$

$= \$\,2300$

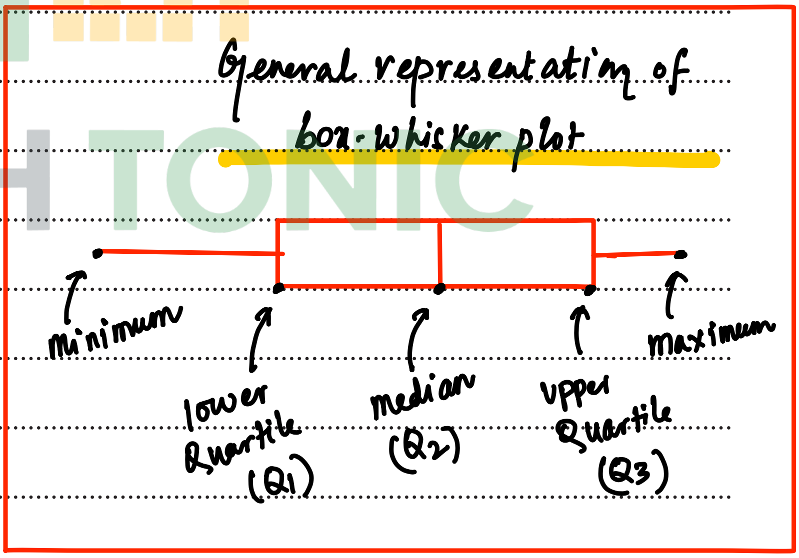
The median salary of the Ravon employees is \$33 800, the lower quartile is \$32 000 and the upper quartile is \$34 400.

(b) Represent the data shown in the back-to-back stem-and-leaf diagram by a pair of box-and-whisker plots in a single diagram. [3]



(c) Comment on whether the mean or the median would be a better representation of the data for the employees at Petral. [1]

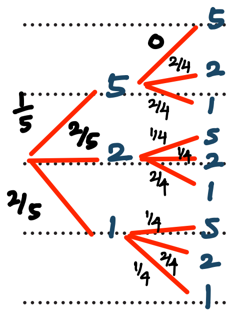
Median would be a better representation of data due to existence of an Outlier. (\$36800)



12.

Jasmine has one \$5 coin, two \$2 coins and two \$1 coins. She selects two of these coins at random. The random variable X is the total value, in dollars, of these two coins.

(a) Show that $P(X = 7) = 0.2$. [1]



$P(X=7) = P(5 \cap 2)$ or $P(2 \cap 5)$

$= \frac{1}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$

$= \frac{4}{20}$

$= 0.2$

(b) Draw up the probability distribution table for X . [3]

possible X-values : 7, 6, 4, 3, 2

$P(X=6) = P(5 \cap 1)$ or $P(1 \cap 5)$

$= \frac{1}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$

$= 0.2$

$P(X=4) = P(2 \cap 2)$

$= \frac{2}{5} \times \frac{1}{4}$

$= 0.1$

$P(X=2) = P(1 \cap 1)$

$= \frac{2}{5} \times \frac{1}{4} = 0.1$

x	2	3	4	6	7
$P(X=x)$	0.1	0.4	0.1	0.2	0.2

$P(X=3) = 1 - (0.1 + 0.1 + 0.2 + 0.2) = 0.4$

(c) Find the value of Var(X). [3]

x	2	3	4	6	7	
$P(X=x)$	0.1	0.4	0.1	0.2	0.2	
xp	0.2	1.2	0.4	1.2	1.4	$\Sigma xp = 4.4$
x^2p	0.4	3.6	1.6	7.2	9.8	$\Sigma x^2p = 22.6$

$$\boxed{\begin{aligned} E(X) &= \Sigma xp \\ E(X^2) &= \Sigma x^2p \end{aligned}}$$

$$\text{Var}(X) = E(x^2) - \{E(X)\}^2$$

$$= 22.6 - 4.4^2$$

$$= 3.24$$

13.

(a) How many different arrangements are there of the 10 letters in the word REGENERATE? [1]

$$\frac{10!}{2! \times 4!} = 75600$$

2R's 4E's

(b) How many different arrangements are there of the 10 letters in the word REGENERATE in which the 4 Es are together and the 2 Rs have exactly 3 letters in between them? [4]

$\boxed{E\ E\ E\ E}$ R _ _ _ R _ 4!

_ R _ _ _ R $\boxed{E\ E\ E\ E}$ 4!

_ $\boxed{E\ E\ E\ E}$ R _ _ _ R 4!

R _ _ _ R $\boxed{E\ E\ E\ E}$ _ 4!

$\boxed{E\ E\ E\ E}$ _ R _ _ _ R 4!

R _ _ _ R _ $\boxed{E\ E\ E\ E}$ 4!

$4! \times 6 = 144$

4E's
2R's
G N A T

(c) Find the probability that a randomly chosen arrangement of the 10 letters in the word REGENERATE is one in which the consonants (G, N, R, R, T) and vowels (A, E, E, E, E) alternate, so that no two consonants are next to each other and no two vowels are next to each other. [5]

A G E N E R E R E T

$$\underbrace{\frac{5!}{2!}}_{2\ R's} \times \underbrace{\frac{5!}{4!}}_{4\ E's}$$

G A N E R E R E T E

$$\underbrace{\frac{5!}{4!}}_{4\ E's} \times \underbrace{\frac{5!}{2!}}_{2\ R's}$$

Total Arrangements : $2 \times \frac{5!}{2!} \times \frac{5!}{4!}$

$= 600$

Probability $= \frac{600}{75600}$

14.

The numbers on the faces of a fair six-sided dice are 1, 2, 2, 3, 3, 3. The random variable X is the total score when the dice is rolled twice.

(a) Draw up the probability distribution table for X. [3]

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

x	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$

(b) Find the value of Var(X). [3]

$E(X)$: $\left(2\times\frac{1}{36}\right)+\left(3\times\frac{4}{36}\right)+\left(4\times\frac{10}{36}\right)+\left(5\times\frac{12}{36}\right)+\left(6\times\frac{9}{36}\right)$

$E(X) = \frac{2}{36}+\frac{12}{36}+\frac{40}{36}+\frac{60}{36}+\frac{54}{36}$

$E(X) = \frac{14}{3}$

$E(X^2)$: $\left(2^2\times\frac{1}{36}\right)+\left(3^2\times\frac{4}{36}\right)+\left(4^2\times\frac{10}{36}\right)+\left(5^2\times\frac{12}{36}\right)+\left(6^2\times\frac{9}{36}\right)$

$E(X^2) = \frac{1}{9}+1+\frac{40}{9}+\frac{25}{3}+9 = \frac{206}{9}$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{206}{9} - \left(\frac{14}{3}\right)^2 = \frac{10}{9}$$

(c) Find the probability that X is even given that $X > 3$. [2]

Conditional probability

$$P(\text{Even} / X > 3) = \frac{P(\text{Even} \cap x > 3)}{P(x > 3)}$$

$$P(\text{Even} \cap x > 3) = P(x = 4) + P(x = 6)$$

$$= \frac{10}{36} + \frac{9}{36}$$

$$= \frac{19}{36}$$

$$P(X > 3) = \frac{10}{36} + \frac{12}{36} + \frac{9}{36}$$

$$= \frac{31}{36}$$

$$P(\text{Even} / x > 3) = \frac{\frac{19}{36}}{\frac{31}{36}}$$

$$= \frac{19}{31}$$

15.

Box A contains 6 green balls and 3 yellow balls.

Box B contains 4 green balls and x yellow balls.

A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

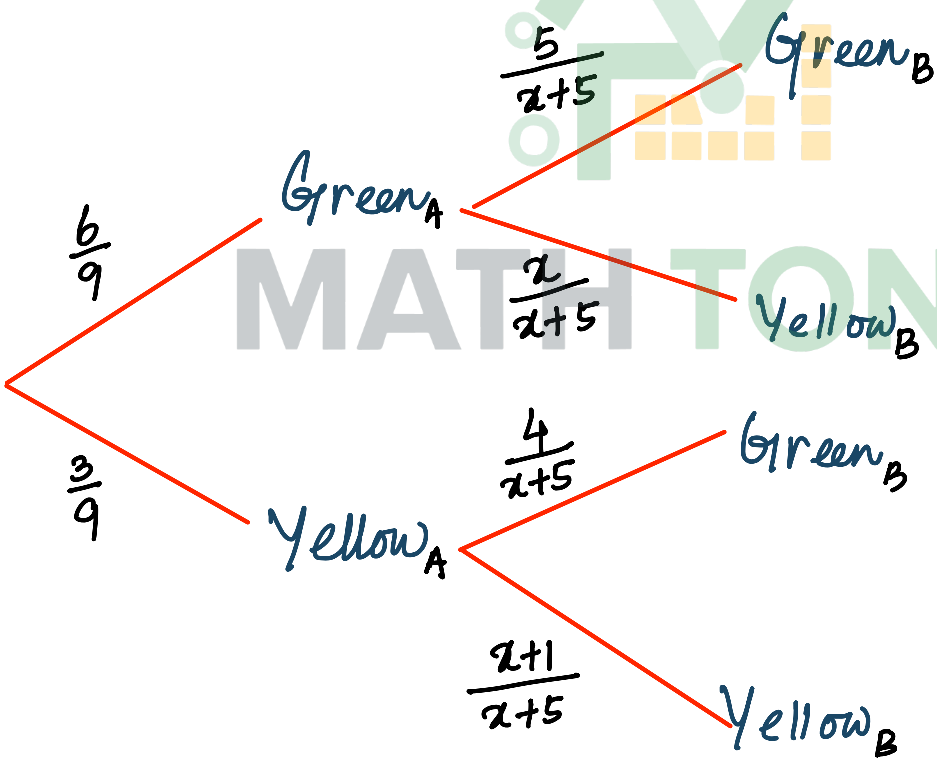
(a) Draw a tree diagram to represent this information, showing the probability on each of the branches. [4]

BOX A :
6 Green 3 Yellow

Total = 9

BOX B
4 Green x Yellow

Total = $4+x$



The probability that both the balls chosen are the same colour is $\frac{8}{15}$.

(b) Find the value of x. [3]

$$P(\text{Same Colour}) = \frac{8}{15}$$

$$P(G_A\, G_B) + P(Y_A\, Y_B) = \frac{8}{15}$$

$$\frac{6}{9} \times \frac{5}{x+5} + \frac{3}{9} \times \frac{x+1}{x+5} = \frac{8}{15}$$

$$\frac{30}{9(x+5)} + \frac{3(x+1)}{9(x+5)} = \frac{8}{15}$$

$$\frac{30 + 3(x+1)}{9(x+5)} = \frac{8}{15}$$

$$\frac{33+3x}{9x+45} = \frac{8}{15}$$

$$15(33+3x) = 8(9x+45)$$

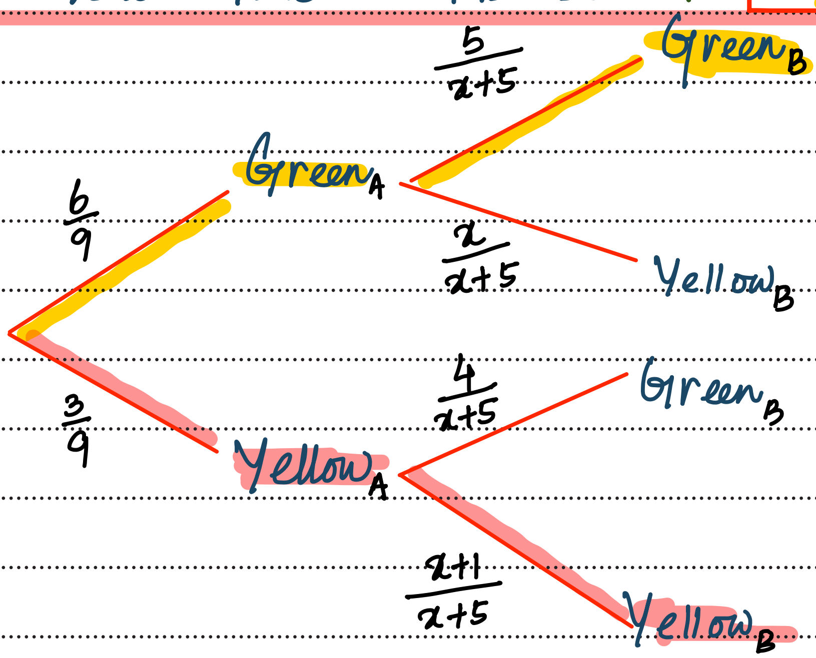
$$495 + 45x = 72x + 360$$

$$72x - 45x = 495 - 360$$

$$27x = 135$$

$$x = \frac{135}{27}$$

$$\boxed{x = 5}$$



16.

The times taken, in seconds, by 15 members of each of two swimming clubs, the Penguins and the Dolphins, to swim 50 metres are shown in the following table.

Penguins	35	39	42	44	45	45	48	50	56	58	59	61	66	68	72
Dolphins	36	41	43	48	49	49	50	51	54	56	56	60	61	64	71

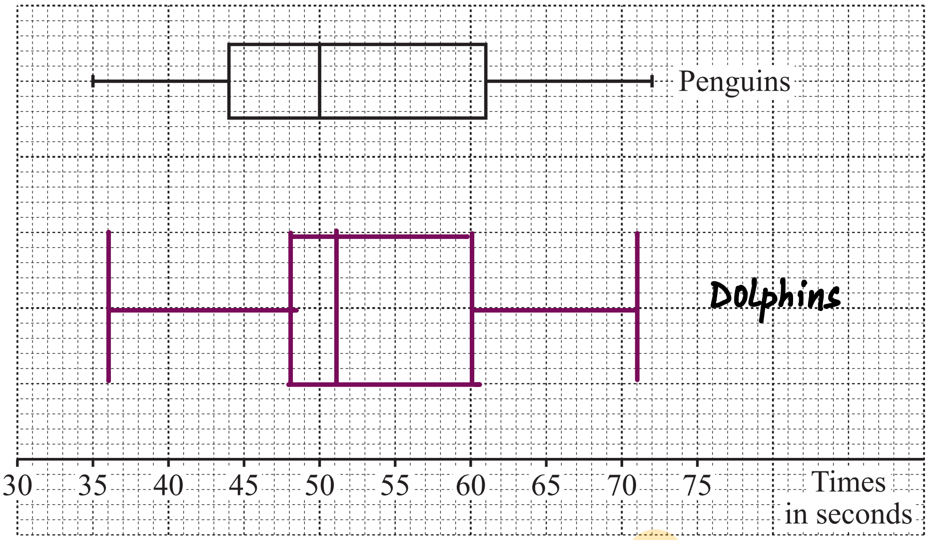
(a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Penguins on the left-hand side. [4]

Penguins		Dolphins
9 5	3	6
8 5 5 4 2	4	1 3 8 9 9
9 8 6 0	5	0 1 4 6 6
8 6 1	6	0 1 4
2	7	1

Key: 2|4|1 means 42 seconds for Penguins and 41 seconds for Dolphins

The diagram shows a box-and-whisker plot representing the times for the Penguins.

(b) On the same diagram, draw a box-and-whisker plot to represent the times for the Dolphins. [3]



Dolphin:

Median $= \frac{1}{2}(15+1)^{th} = 8^{th}$ value $= 51$

$Q_1 = \frac{1}{4}[15+1]^{th} = 4^{th}$ value $= 48$

$Q_3 = \frac{3}{4}[15+1]^{th} = 12^{th}$ value $= 60$

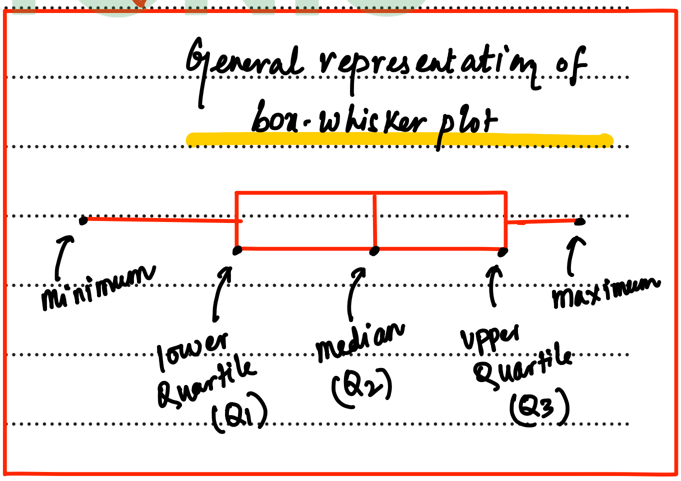
(c) Hence state **one** difference between the distributions of the times for the Penguins and the Dolphins. [1]

IQR (Penguin)

$61 - 44 = 17$

IQR (Dolphin)

$60 - 48 = 12$



Time taken by Penguins are more spread

Compared to Dolphins.

17.

(a) How many different arrangements are there of the 9 letters in the word RECORDERS? [1]

$$\frac{9!}{3! \times 2!} = 30240$$

3R's 2E's

(b) How many different arrangements are there of the 9 letters in the word RECORDERS in which there is an E at the beginning, an E at the end and the three Rs are not all together? [3]

E's at each end and R's together:

E $\boxed{R\ R\ R}$ _ _ _ _ E

5!

E's at each end and no other restriction:

E _ _ _ _ _ _ _ E

$$\frac{7!}{3!}$$

3R's

3R's not together: $\frac{7!}{3!} - 5!$

$= 720$

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The 9 letters of the word RECORDERS are divided at random into two groups: a group of 5 letters and a group of 4 letters.

(c) Find the probability that the three Rs are in the same group. [4]

Group of 5 letters   Group of 4 letters

R R R $\boxed{\_\ \_}$ $\boxed{\_\ \_\ \_\ \_}$

${}^6C_2$ ${}^4C_4$

or

$\boxed{\_\ \_\ \_\ \_\ \_}$ R R R $\boxed{\_}$

${}^5C_5$ ${}^6C_1$

Probability : $\dfrac{{}^6C_2 \times {}^4C_4 + {}^5C_5 \times {}^6C_1}{{}^9C_5 \times {}^4C_4}$

$= \dfrac{21}{126} = \dfrac{1}{6}$

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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