1 A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

(b) Given that $\mathrm{E}(X) = 0.25$, find the value of $\mathrm{Var}(X)$. [2]

2.

A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

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(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

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The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

3.

(a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

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(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

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4.

There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

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(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

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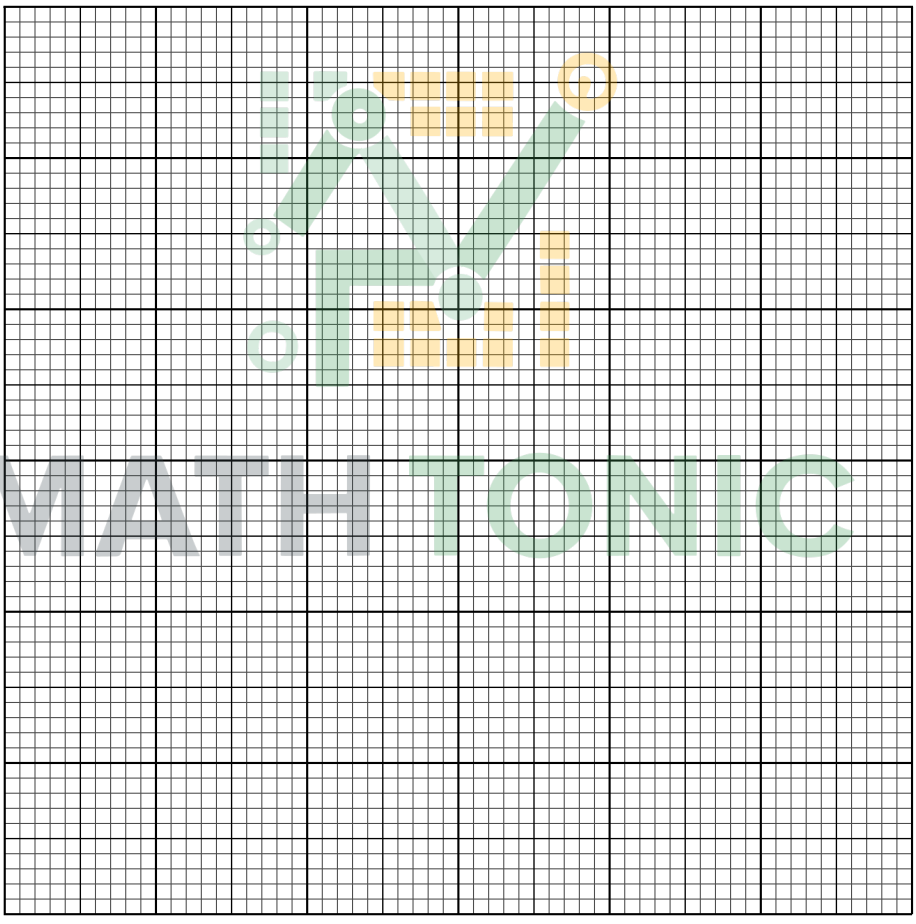
5.

The times taken to travel to college by 2500 students are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 90$
Frequency	440	720	920	300	120

(a) Draw a histogram to represent this information. [4]



From the data, the estimate of the mean value of t is 31.44.

(b) Calculate an estimate of the standard deviation of the times taken to travel to college. [3]

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(c) In which class interval does the upper quartile lie? [1]

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It was later discovered that the times taken to travel to college by two students were incorrectly recorded. One student's time was recorded as 15 instead of 5 and the other's time was recorded as 65 instead of 75.

(d) Without doing any further calculations, state with a reason whether the estimate of the standard deviation in part **(b)** would be increased, decreased or stay the same. [1]

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6.

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

(b) Find $\mathrm{E}(X)$. [1]

7.

Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

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(b) Find the probability that Janice finishes the game. [3]

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(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

8.

For n values of the variable x, it is given that

$$\Sigma(x - 200) = 446 \quad \text{and} \quad \Sigma x = 6846.$$

Find the value of n. [3]

9.

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

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(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

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(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

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(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

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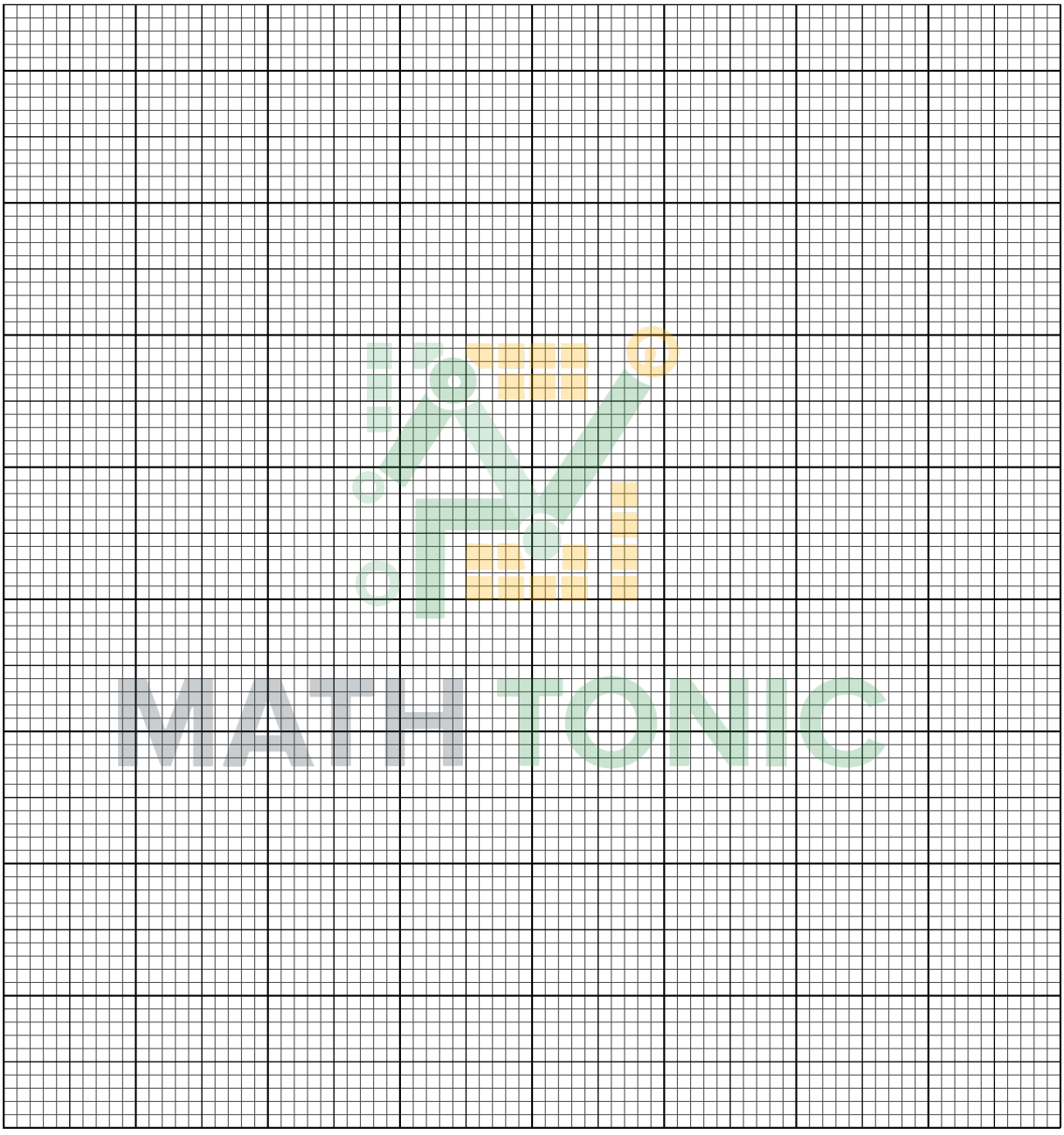
10.

The time taken, t minutes, to complete a puzzle was recorded for each of 150 students. These times are summarised in the table.

Time taken (t minutes)	$t \leqslant 25$	$t \leqslant 50$	$t \leqslant 75$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$
Cumulative frequency	16	44	86	104	132	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 20th percentile of the data. [1]

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11.

The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

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(b) Find $E(X)$ and $Var(X)$. [3]

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12.

Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

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On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

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13.

The probability distribution table for a random variable X is shown below.

x	-2	-1	0.5	1	2
$P(X = x)$	0.12	p	q	0.16	0.3

Given that $E(X) = 0.28$, find the value of p and the value of q. [4]

14.

A game is played with an ordinary fair 6-sided die. A player throws the die once. If the result is 2, 3, 4 or 5, that result is the player's score and the player does not throw the die again. If the result is 1 or 6, the player throws the die a second time and the player's score is the sum of the two numbers from the two throws.

(a) Draw a fully labelled tree diagram to represent this information. [2]

Events A and B are defined as follows.

A: the player's score is 5, 6, 7, 8 or 9

B: the player has two throws

(b) Show that $P(A) = \frac{1}{3}$. [3]

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(c) Determine whether or not events A and B are independent. [2]

(d) Find $\mathrm{P}(B \mid A')$. [3]

15.

A Social Club has 15 members, of whom 8 are men and 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. [4]

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? [3]

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In one photograph Abel, Betty, Cally, Doug, Eve, Freya and Gino are the 7 members in the back row.

(c) In how many different ways can these 7 members be arranged so that Abel and Betty are next to each other and Freya and Gino are not next to each other? [3]

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16.

Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $P(B) = \frac{7}{16}$. [2]

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(b) Find $P(A \mid B)$. [2]

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The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw up the probability distribution table for X. [3]

17.

(a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

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(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

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(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

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18.

50 values of the variable x are summarised by

$$\Sigma(x - 20) = 35 \qquad \text{and} \qquad \Sigma x^2 = 25\,036.$$

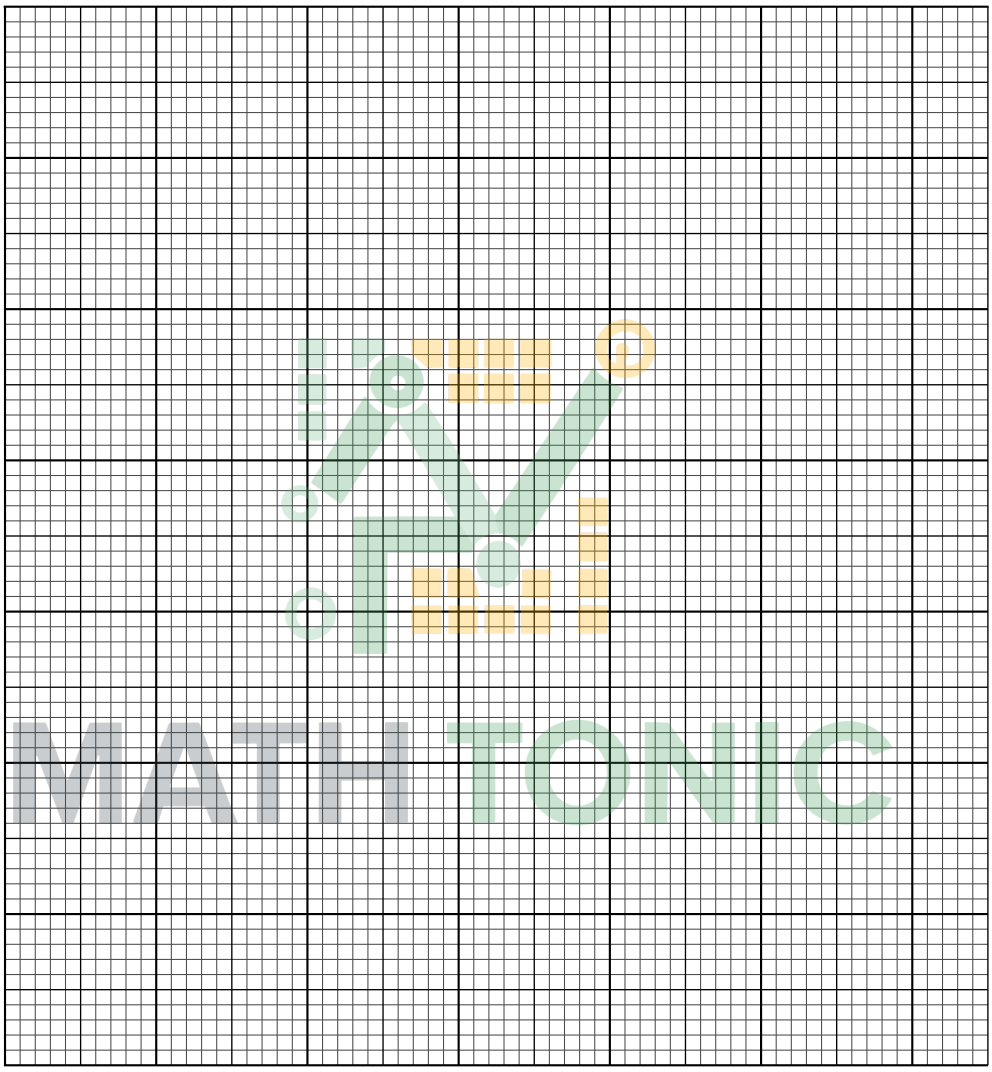
Find the variance of these 50 values. [3]

19.

The times, t minutes, taken to complete a walking challenge by 250 members of a club are summarised in the table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 60$
Cumulative frequency	32	66	112	178	228	250

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 60th percentile of the data. [1]

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It is given that an estimate for the mean time taken to complete the challenge by these 250 members is 34.4 minutes.

(c) Calculate an estimate for the standard deviation of the times taken to complete the challenge by these 250 members. [4]

20.

Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game.

Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on his first turn. [2]

(b) Find the probability that Tom wins the game on his second turn. [4]

(c) Find the probability that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. [2]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.