1 A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

	1	2	2	3
−3	−2	−1	−1	0
−2	−1	0	0	1
−1	0	1	1	2
−1	0	1	1	2

Probability Distribution table:

x	−2	−1	0	1	2
$P(X=x)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{2}{1}$

(b) Given that $E(X) = 0.25$, find the value of $Var(X)$. [2]

x	−2	−1	0	1	2
$P(X=x)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{2}{16}$
x^2	4	1	0	1	4
x^2p	$4/16$	$3/16$	0	$5/16$	$8/16$

$\Sigma x^2p = \frac{20}{16}$

$E(x^2) = x^2p$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \frac{20}{16} - (0.25)^2$$

$$\boxed{Var(x) = \frac{19}{16}}$$

2.

A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

3 boys   4 girls   5 adults

No of ways : $^5C_1 \times {}^7C_4$

1 from adult   4 from Remaining 7.

$= 5 \times 35$

$= 175$

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

Possibilites :

Boys (3)	Girls (4)	Adults (5)	
2	1	2	$^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$
2	2	1	$^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$
3	1	1	$^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$
3	2	0	$^3C_3 \times {}^4C_2 \times {}^5C_0 = 6$
2	3	0	$^3C_2 \times {}^4C_3 \times {}^5C_0 = 12$

Total No. of ways :

$120 + 90 + 20 + 6 + 12$

$= 248$

The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

A = Adult B = Boys

A_1 $\boxed{B_1 \; B_2 \; B_3}$ ___ ___ ___ ___ ___ ___ ___ A_2

(Internal Arrangement of 3 boys)

$8! \times 3! \times {}^5P_2$

Picking 2 adults from 5

$= 4838400$

3.

(a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

$\boxed{D \; D}$ $\boxed{E \; E \; E}$ _ _ _

✓ ✓ ✓ ✓ ✓

No of arrangements : $5!$

$= 120$

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

No of arrangement (with 3E's together)

$\boxed{E \; E \; E}$ _ _ _ _ _

✓ ✓ ✓ ✓ ✓ ✓

$$\frac{6!}{2!} = 360$$

for repeating 2A's

No of arrangement without restriction :

$$\frac{8!}{2! \times 3!} = 3360$$

2 D's 2 E's

No of arrangements (3E's not together) :

$3360 - 360 = 3000$

4.

There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

MEN(6) WOMEN(8)

$14-2=12$

No of ways: ${}^{12}C_4$
(Mr. Lan chosen already)

No of ways: ${}^{12}C_4$
(Mrs. Lan chosen already)

Total: $2 \times {}^{12}C_4$

$\boxed{990}$

Alternative way:

without restriction $= {}^{14}C_5$

Both chosen already $= {}^{12}C_3$

None of them chosen $= {}^{12}C_5$

Total $= {}^{14}C_5 - ({}^{12}C_3 + {}^{12}C_5)$

$\boxed{= 990}$

(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

MEN (6) WOMEN (8) Mrs. Lan already chosen (so women will be selected from 7)

1	4 (3+1)	${}^6C_1 \times {}^7C_3$	$= 210$
2	3 (2+1)	${}^6C_2 \times {}^7C_2$	$= 315$
0	5 (4+1)	${}^6C_0 \times {}^7C_4$	$= 35$

Total No. ways: $210 + 315 + 35$

$= 560$

5.

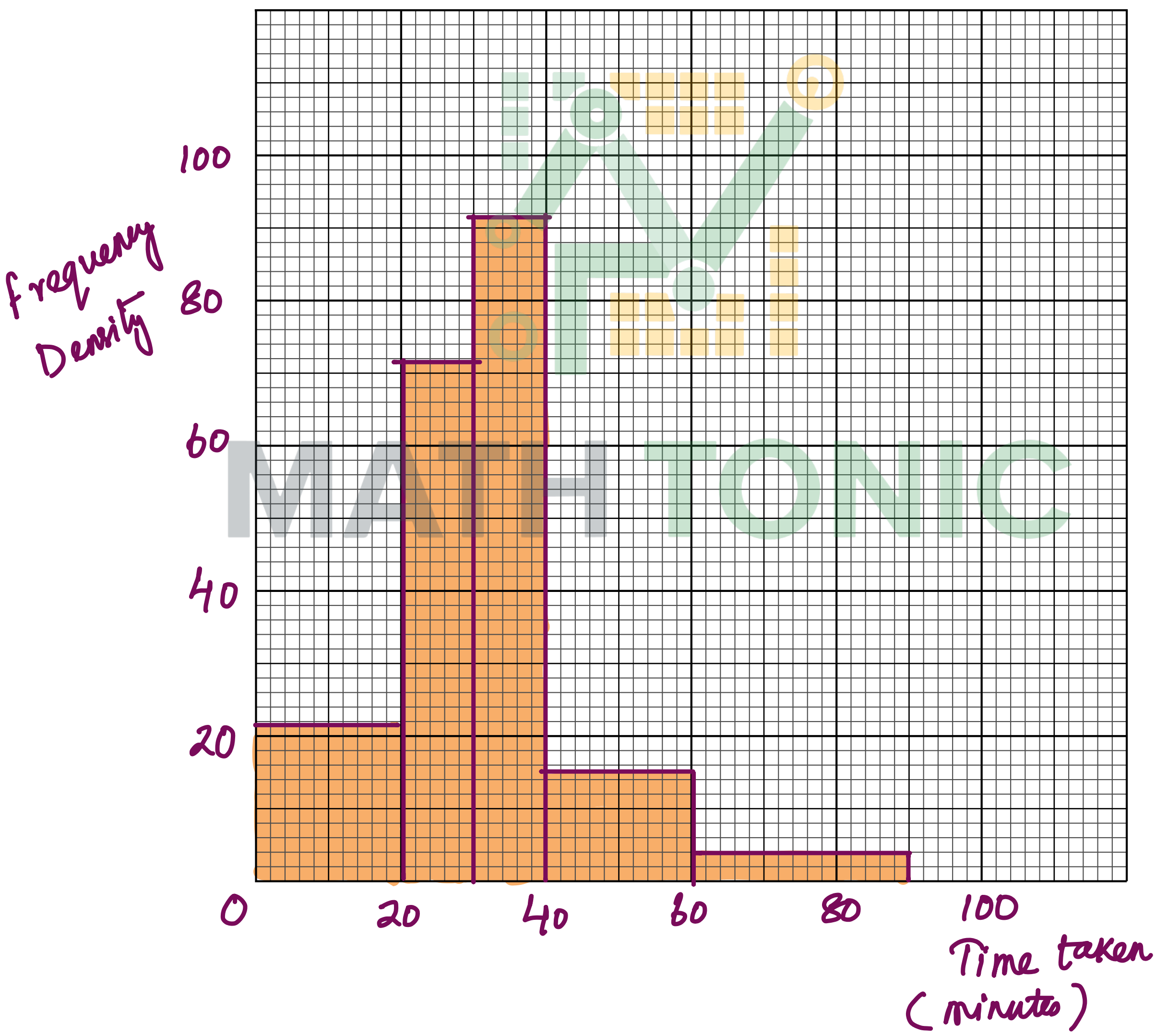
The times taken to travel to college by 2500 students are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 90$
Frequency	440	720	920	300	120

(a) Draw a histogram to represent this information. [4]

Class width	20	10	10	20	30
Frequency Density	22	72	92	15	4

$$F \cdot D = \frac{\text{Frequency}}{\text{Class width}}$$



From the data, the estimate of the mean value of t is 31.44.

(b) Calculate an estimate of the standard deviation of the times taken to travel to college. [3]

Time (t)	Mid Point (x)	f	fx^2
$0 \leq t < 20$	10	440	44000
$20 \leq t < 30$	25	720	450000
$30 \leq t < 40$	35	920	1127000
$40 \leq t < 60$	50	300	750000
$60 \leq t < 90$	75	120	675000

Standard deviation

$$= \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

$$\sqrt{\frac{3046000}{2500} - (31.44)^2}$$

$\Sigma f x^2 = 3046000$ $\quad$ $15.163 = 15.2$

(c) In which class interval does the upper quartile lie? [1]

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 90$
Frequency	440	720	920	300	120
Cumulative freq.	440	1160	2080	2380	2500

1875th value lies here

Upper Quartile: $\frac{3}{4} \times (2500) = 1875^{th}$ value

$\boxed{\text{Class interval: } 30 \leq t < 40}$

It was later discovered that the times taken to travel to college by two students were incorrectly recorded. One student's time was recorded as 15 instead of 5 and the other's time was recorded as 65 instead of 75.

(d) Without doing any further calculations, state with a reason whether the estimate of the standard deviation in part **(b)** would be increased, decreased or stay the same. [1]

Standard deviation stays the same, because the data recorded is in the same intervals.

$15 \xrightarrow{-10} 5$ $\qquad$ $+10$

$75 \xrightarrow{+10} 65$ $\qquad$ -10

No change

6.

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

F = fair coin
B = Biased coin

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

For Biased coin

$P(H) = \frac{7}{10}$ $\quad P(T) = 1 - \frac{7}{10} = \frac{3}{10}$

1 H

F	F	F	B
H	T	T	T
T	H	T	T
T	T	H	T
T	T	T	H

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{10} = \frac{3}{80}$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{10} = \frac{7}{80}$

$P(1H) = \left(3 \times \frac{3}{80}\right) + \frac{7}{80} = \frac{1}{5}$

$\boxed{a = \frac{1}{5}}$

2 H

F	F	F	B
H	H	T	T
H	T	H	T
T	H	H	T
T	H	T	H
H	T	T	H
T	T	H	H

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{10} = \frac{3}{80}$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{10} = \frac{7}{80}$

$P(2H) = \left(3 \times \frac{3}{80}\right) + \left(3 \times \frac{7}{80}\right) = \frac{3}{8}$

$\boxed{b = \frac{3}{8}}$

$P(3H) = 1 - \left[\frac{3}{80} + \frac{1}{5} + \frac{3}{8} + \frac{7}{80}\right]$

$P(3H) = \frac{3}{10}$

$\boxed{c = \frac{3}{10}}$

(b) Find $E(X)$. [1]

$E(X) = \left(0 \times \frac{3}{80}\right) + \left(1 \times \frac{1}{5}\right) + \left(2 \times \frac{3}{8}\right) + \left(3 \times \frac{3}{10}\right) + \left(4 \times \frac{7}{80}\right)$

$E(X) = 0 + \frac{1}{5} + \frac{6}{8} + \frac{9}{10} + \frac{28}{80}$

$\boxed{E(X) = \frac{11}{5}}$

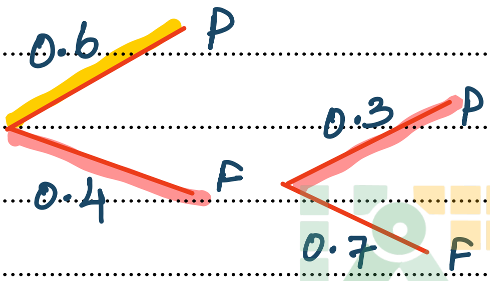
7.

Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

Level 1:



$P(P) + P(FP)$

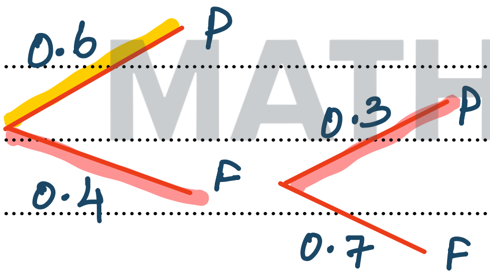
$0.6 + 0.4 \times 0.3$

$0.6 + 0.12$

0.72

(b) Find the probability that Janice finishes the game. [3]

Level 1



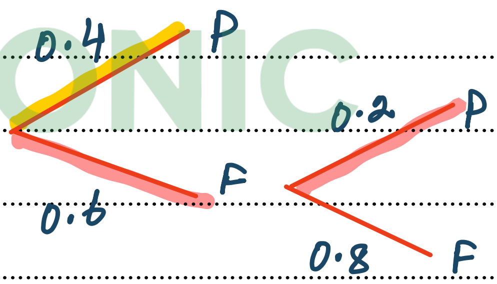
$P(P_1) + P(F_1P_1)$

$0.6 + 0.4 \times 0.3$

$0.6 + 0.12$

0.72

Level 2



$P(P_2) + P(F_2P_2)$

$0.4 + (0.6 \times 0.2)$

0.52

Probability that Janice finishes the game:

0.72×0.52

0.3744

(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

$P(\text{fails one attempt} \mid \text{finishes the game})$ Conditional probability

$$= \frac{P(\text{Fails one Attempt} \cap \text{finishes the game})}{P(\text{finishes the game})}$$

$$= \frac{P(P_1 F_2 P_2) + P(F_1 P_1 P_2)}{0.3744}$$ from part b

$$= \frac{(0.6 \times 0.6 \times 0.2) + (0.4 \times 0.3 \times 0.4)}{0.3744}$$

$$= \frac{0.12}{0.3744}$$

$$= 0.321$$

8.

For n values of the variable x, it is given that

$$\Sigma(x - 200) = 446 \quad \text{and} \quad \Sigma x = 6846.$$

Find the value of n. [3]

$\Sigma(x - 200) = 446$

$\Sigma x - 200n = 446$

$\Sigma x = 446 + 200n$

$6846 = 446 + 200n$

$200n = 6846 - 446$

$200n = 6400$

$n = \frac{6400}{200}$

$\boxed{n = 32}$

9.

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

$$\frac{9!}{2! \times 2!} = 90720$$

2C's 2O's

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

No of arrangements (2 O's together):

C $\boxed{O \quad O}$ _ _ _ _ _ C

✓ ✓ ✓ ✓ ✓ ✓

$6!$

No of arrangement only c's at each end

$$\frac{7!}{2!}$$

2 O's repeating

No of arrangement with Cs at each end and 2 O's not together: $\frac{7!}{2!} - 6!$

$= 2520 - 720$

$= 1800$

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

Possibilities:

C(2)	O(2)	Others(5)		
1	2	1	C O O _	${}^5C_1 = 5$
2	1	1	C C O _	${}^5C_1 = 5$
1	0	3	C _ _ _	${}^5C_3 = 10$
0	1	3	O _ _ _	${}^5C_3 = 10$
2	0	2	C C _ _	${}^5C_2 = 10$
0	2	2	O O _ _	${}^5C_2 = 10$

Total $= 5+5+10+10+10+10$

$= 50$

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

Possibilities:

C O _ | C O _ | _ _ _ $\quad \dfrac{{}^5C_1 \times {}^4C_1 \times {}^3C_3}{2!} = 10$

for repitition of selection CO CO

C O O | C _ _ | _ _ _ $\quad {}^5C_2 \times {}^3C_3 = 10$

C O _ | O _ _ | C _ _ $\quad {}^5C_1 \times {}^4C_2 \times {}^2C_2 = 30$

O O _ | C _ _ | C _ _ $\quad \dfrac{{}^5C_1 \times {}^4C_2 \times {}^2C_2}{2!} = 15$

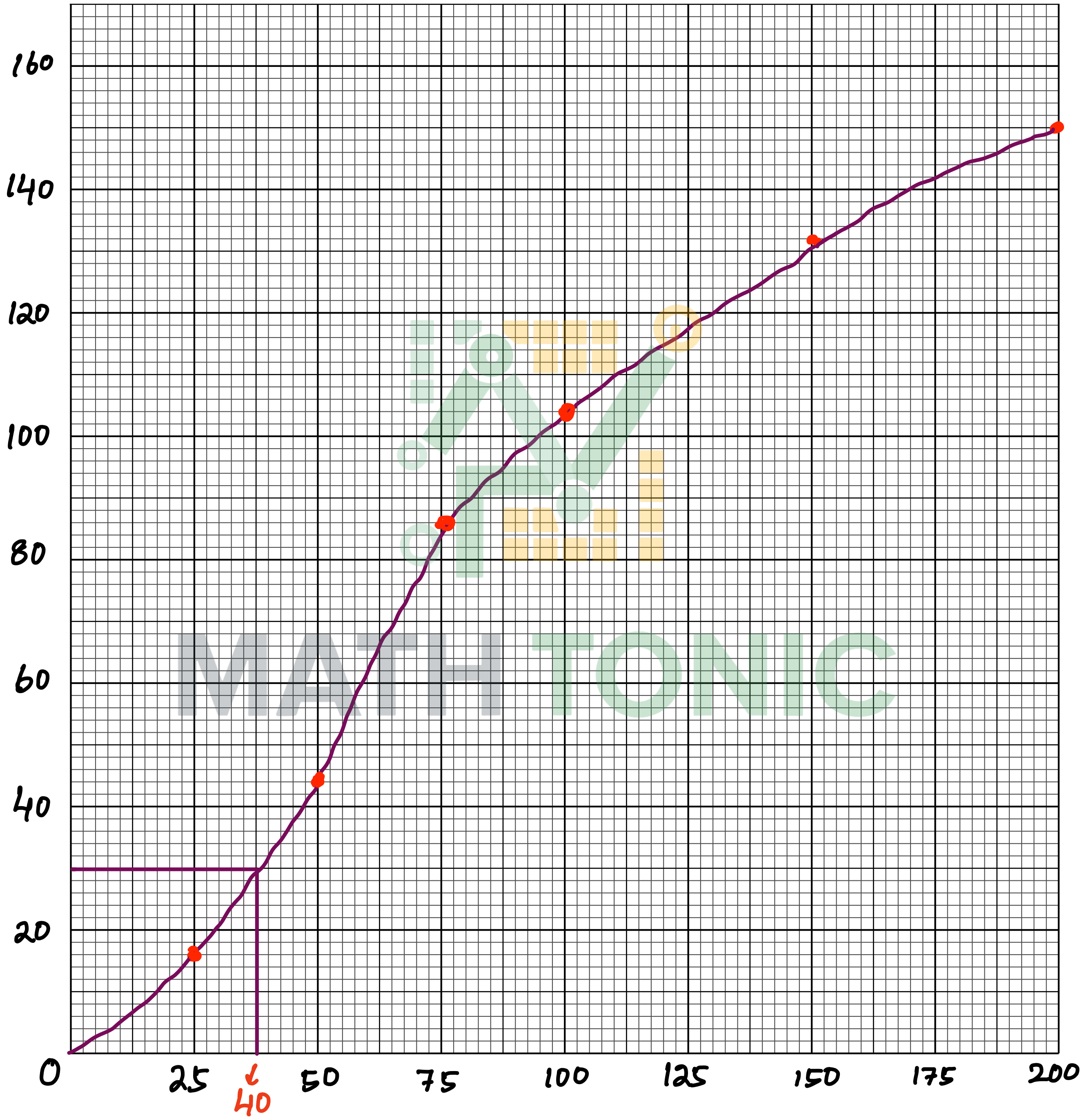
Total: $10+10+30+15 = \boxed{65}$ for repetition of selection C__ C__

10.

The time taken, t minutes, to complete a puzzle was recorded for each of 150 students. These times are summarised in the table.

Time taken (t minutes)	$t \leqslant 25$	$t \leqslant 50$	$t \leqslant 75$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$
Cumulative frequency	16	44	86	104	132	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 20th percentile of the data. [1]

$\frac{20}{100} \times 150 = 30^{th}$ value $= 40$

11.

The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$P(X=x) = kx^2$

$x=-2 \quad P(x=-2) = k\times(-2)^2 = 4k \qquad 4\times\frac{1}{8}=\frac{4}{8}$

$x=1 \quad P(x=1) = k\times 1^2 = k \qquad \frac{1}{8}$

$x=2 \quad P(x=2) = k\times 2^2 = 4k \qquad 4\times\frac{1}{8}=\frac{4}{8}$

$x=3 \quad P(x=3) = k\times 3^2 = 9k \qquad 9\times\frac{1}{8}=\frac{9}{8}$

$\Sigma P = 1$

$4k+k+4k+9k = 1$

$18k = 1$

$k = \frac{1}{18}$

x	-2	1	2	3
$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

(b) Find $E(X)$ and $Var(X)$. [3]

x	-2	1	2	3
$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$
xp	$\frac{-8}{18}$	$\frac{1}{18}$	$\frac{8}{18}$	$\frac{27}{18}$
x^2p	$\frac{16}{18}$	$\frac{1}{18}$	$\frac{16}{18}$	$\frac{81}{18}$

$\Sigma xp = \frac{28}{18} = \frac{14}{9}$

$\Sigma x^2p = \frac{114}{18}$

$E(X) = \Sigma xp \qquad E(X^2) = \Sigma x^2 p$

$Var(X) = E(x^2) - [E(x)]^2$

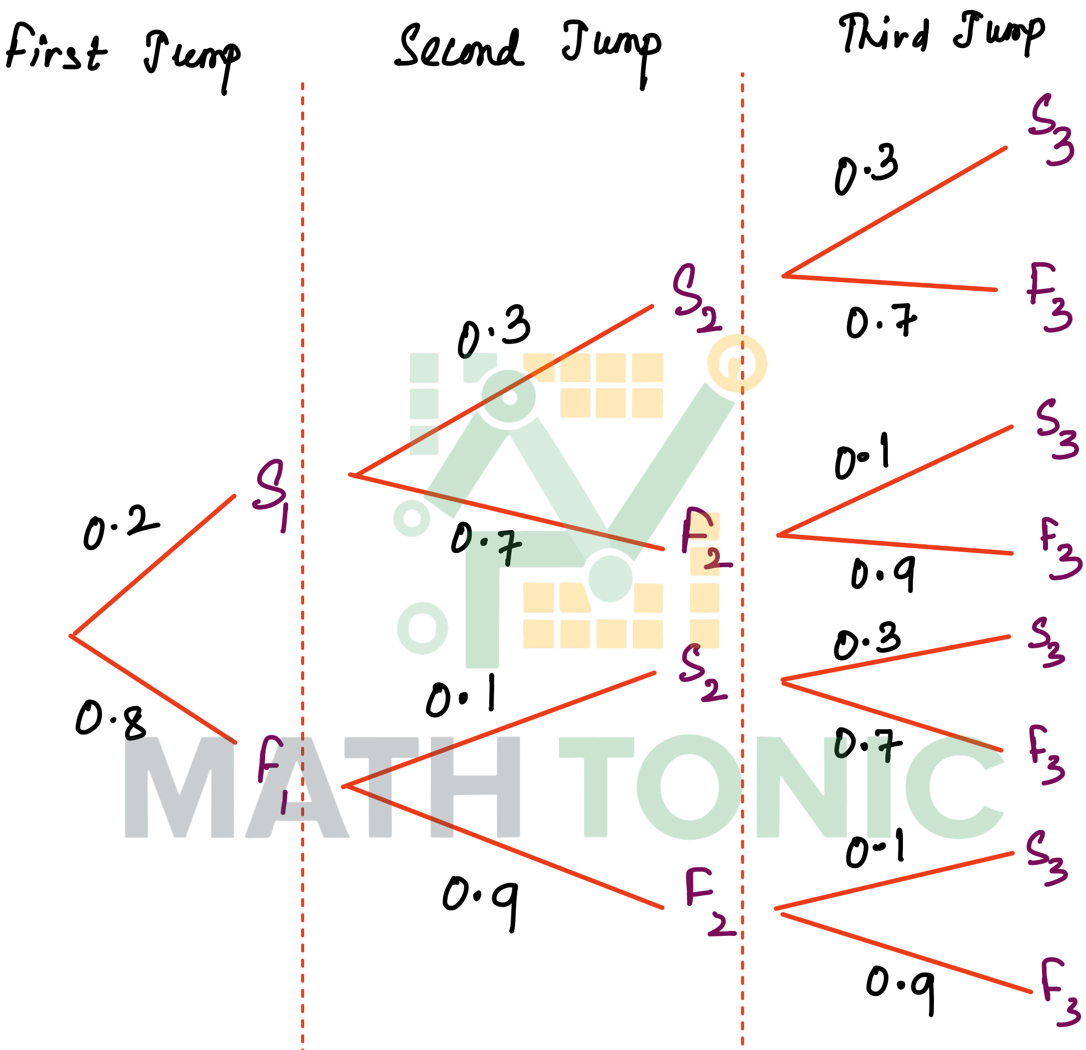
$= \frac{114}{18} - \left(\frac{14}{9}\right)^2$

$= \frac{114}{18} - \frac{196}{81} = \frac{317}{81}$

12.

Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]



(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

$$P(x=1 \mid x\geq 1) = \frac{P(x=1 \cap x\geq 1)}{P(x\geq 1)}$$

$$= \frac{P(x=1)}{1-P(x<1)}$$

$$= \frac{P(S_1F_2F_3)+P(F_1S_2F_3)+P(F_1F_2S_3)}{1-P(F_1F_2F_3)}$$

$$= \frac{(0.2\times0.7\times0.9)+(0.8\times0.1\times0.7)+(0.8\times0.9\times0.1)}{1-(0.8\times0.9\times0.9)}$$

$$= \frac{0.254}{0.352} = \frac{127}{176}$$

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

$P(1^{st}\ 3\ \text{Success}) = P(SSSFFF)$

$= 0.2\times0.3\times0.3\times0.7\times0.9\times0.9$

$= 0.010206$

$P(\text{Last } 3\ \text{Success}) = P(FFFSSS)$

$= 0.8\times0.9\times0.9\times0.1\times0.3\times0.3$

$= 0.005832$

Total probability $= 0.010206 + 0.005832$

$= 0.16038$

$= 0.16$

13.

The probability distribution table for a random variable X is shown below.

x	-2	-1	0.5	1	2
$P(X = x)$	0.12	p	q	0.16	0.3

Given that $E(X) = 0.28$, find the value of p and the value of q. [4]

$\Sigma P = 1$

$0.12 + p + q + 0.16 + 0.3 = 1$

$p + q + 0.58 = 1$

$p + q = 1 - 0.58$

$p + q = 0.42$ —— (i)

$E(x) = \Sigma x p$

$0.28 = (-2 \times 0.12) + (-1 \times P) + (0.5 \times q) + (1 \times 0.16) + (2 \times 0.3)$

$0.28 = -0.24 - P + 0.5q + 0.16 + 0.6$

$-P + 0.5q + 0.52 = 0.28$

$-P + 0.5q = -0.24$ —— (ii)

$\cancel{p} + q = 0.42$

$-\cancel{P} + 0.5q = -0.24$

$1.5q = 0.18$

$q = \dfrac{0.18}{1.5}$

$\boxed{q = 0.12}$

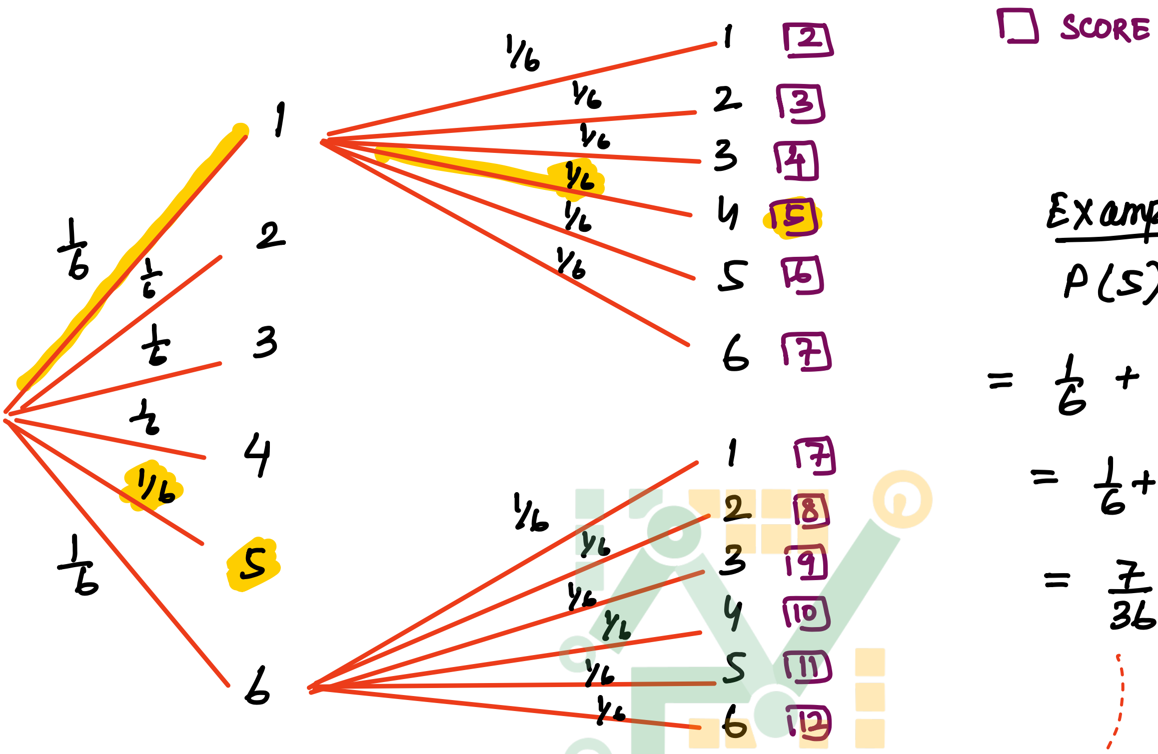
$p + q = 0.42$

$p + 0.12 = 0.42$

$\boxed{p = 0.3}$

14. A game is played with an ordinary fair 6-sided die. A player throws the die once. If the result is 2, 3, 4 or 5, that result is the player's score and the player does not throw the die again. If the result is 1 or 6, the player throws the die a second time and the player's score is the sum of the two numbers from the two throws.

(a) Draw a fully labelled tree diagram to represent this information. [2]



Example:

$P(5)$

$= \frac{1}{6} + \left(\frac{1}{6}\times\frac{1}{6}\right)$

$= \frac{1}{6} + \frac{1}{36}$

$= \frac{7}{36}$

Events A and B are defined as follows.

A: the player's score is 5, 6, 7, 8 or 9

B: the player has two throws

(b) Show that $\mathrm{P}(A) = \frac{1}{3}$. [3]

$P(A) = P(5) + P(6) + P(7) + P(8) + P(9)$

$= \left(\frac{1}{6} + \frac{1}{36}\right) + \left(\frac{1}{36}\right) + \left(\frac{1}{36} + \frac{1}{36}\right) + \frac{1}{36} + \frac{1}{36}$

$= \frac{12}{36}$

$= \frac{1}{3}$

(c) Determine whether or not events A and B are independent. [2]

If A and B are Independent then

$$P(A) \times P(B) = P(A \cap B)$$

To have 2 throws, the player must have got 1 or 6 in the first throw. Therefore

$$P(B) = P(1) + P(6)$$
$$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$P(A \cap B) = P(1 \cap 4,5,6) \text{ or } P(6 \cap 1,2,3)$$
$$= \left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right) + \left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)$$
$$= \frac{6}{36} = \frac{1}{36}$$

$P(A) \times P(B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \neq P(A \cap B)$, Not independent

(d) Find $P(B \mid A')$. [3]

	B	B'
A	$A \cap B$	$A \cap B'$
A'	$A' \cap B$	$A' \cap B'$

$$P(B) = P(A \cap B) + P(A' \cap B)$$
$$P(A' \cap B) = P(B) - P(A \cap B)$$

$$P(B / A') = \frac{P(A' \cap B)}{P(A')}$$

$$= \frac{P(B) - P(A \cap B)}{1 - P(A)}$$

$$= \frac{\frac{1}{3} - \frac{1}{36}}{1 - \frac{1}{3}} = \boxed{\frac{1}{4}}$$

15.

A Social Club has 15 members, of whom 8 are men and 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. [4]

MEN (8)	WOMEN (7)		
5	0	$^8C_5 \times {^7C_0}$	$= 56$
4	1	$^8C_4 \times {^7C_1}$	$= 490$
3	2	$^8C_3 \times {^7C_2}$	$= 1176$

No of ways: $56 + 490 + 1176$

$= 1722$

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? [3]

__ __ __ 3 people $15-3=12$

__ __ __ __ __ 5 people $12-5=7$

__ __ __ __ __ __ __ 7 people

No of ways : ${}^{15}C_3 \times {}^{12}C_5 \times {}^{7}C_7$

$= 455 \times 792 \times 1$

$= 360360$

In one photograph Abel, Betty, Cally, Doug, Eve, Freya and Gino are the 7 members in the back row.

(c) In how many different ways can these 7 members be arranged so that Abel and Betty are next to each other and Freya and Gino are not next to each other? [3]

Abel and betty next to each other :

$\boxed{A \quad B}$ __ __ __ __ __

✓ ✓ ✓ ✓ ✓ ✓

$6! \times 2!$

Abel and betty next to each other and Friya & Gino together.

$\boxed{A \quad B}$ $\boxed{F \quad G}$ __ __ __

✓ ✓ ✓ ✓ ✓

$5! \times 2! \times 2!$

Abel & Betty and Freya & Gino not together : $(6! \times 2!) - (5! \times 2! \times 2!)$

$= 1440 - 480$

$= 960$

16.

Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $P(B) = \frac{7}{16}$. [2]

F	B_1	B_2	
H	H	H	$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$
H	H	T	$\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$
H	T	H	$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$
T	H	H	$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$
T	T	H	$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$
T	H	T	$\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$

$$P(B) = \frac{1}{32} + \frac{3}{32} + \frac{3}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32} = \frac{14}{32} = \frac{7}{16}$$

(b) Find $P(A \mid B)$. [2]

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = \frac{1}{32}$

H H H

$P(B) = \frac{7}{16}$

from part(a)

$$P(A|B) = \frac{\frac{1}{32}}{\frac{7}{16}}$$

$$P(A|B) = \frac{1}{14}$$

The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw up the probability distribution table for X. [3]

	F	B_1	B_2	
3H	H	H	H	$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$
2H	H	H	T	$\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$
	H	T	H	$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$
	T	H	H	$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$
1H	T	T	H	$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$
	T	H	T	$\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$
	H	T	T	$\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32}$
0H	T	T	T	$\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32}$

2H: $7/32$

1H: $15/32$

Probability Distribution table:

x	0	1	2	3
$P(X=x)$	$9/32$	$15/32$	$7/32$	$1/32$

17.

(a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

$\boxed{A \ A}$ $\boxed{L \ L}$ _ _ _ _ _

✓ ✓ ✓ ✓ ✓ ✓ ✓

No of arrangements : $7! = 5040$

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

A $\boxed{L \ L}$ _ _ _ _ A _ ← I, G, T, O, R

✓ ✓ ✓ ✓ ✓ 5 ways

$5! \times 5$

_ A $\boxed{L \ L}$ _ _ _ _ A

5 ways I, G, T, O, R ✓ ✓ ✓ ✓ ✓

$5! \times 5$

Total No of ways : $2 \times 5! \times 5 = 1200$

Total No of ways (without restriction) : $\frac{9!}{2! \times 2!}$ $= 90720$

2A's

2L's

Probability : $\frac{1200}{90720} = \frac{5}{378}$

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

A	L	Others (5)		
1	0	4	A $\boxed{\_\ \_\ \_\ \_}$	${}^5C_4 = 5$
1	1	3	A L $\boxed{\_\ \_\ \_}$	${}^5C_3 = 10$
2	0	3	A A $\boxed{\_\ \_\ \_}$	${}^5C_3 = 10$
2	1	2	A A L $\boxed{\_\ \_}$	${}^5C_2 = 10$

Total number of Selections:

$5 + 10 + 10 + 10 = 35$

18.

50 values of the variable x are summarised by

$$\Sigma(x-20) = 35 \qquad \text{and} \qquad \Sigma x^2 = 25\,036.$$

Find the variance of these 50 values. [3]

$\Sigma(x-20) = 35$

$\Sigma x - 20n = 35$

$\Sigma x - 20\times 50 = 35$

$\Sigma x = 35 + 1000$

$\Sigma x = 1035$

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{1035}{50} = 20.7$

Variance : $\dfrac{\Sigma x^2}{n} - (\bar{x})^2$

$\dfrac{25036}{50} - (20.7)^2$

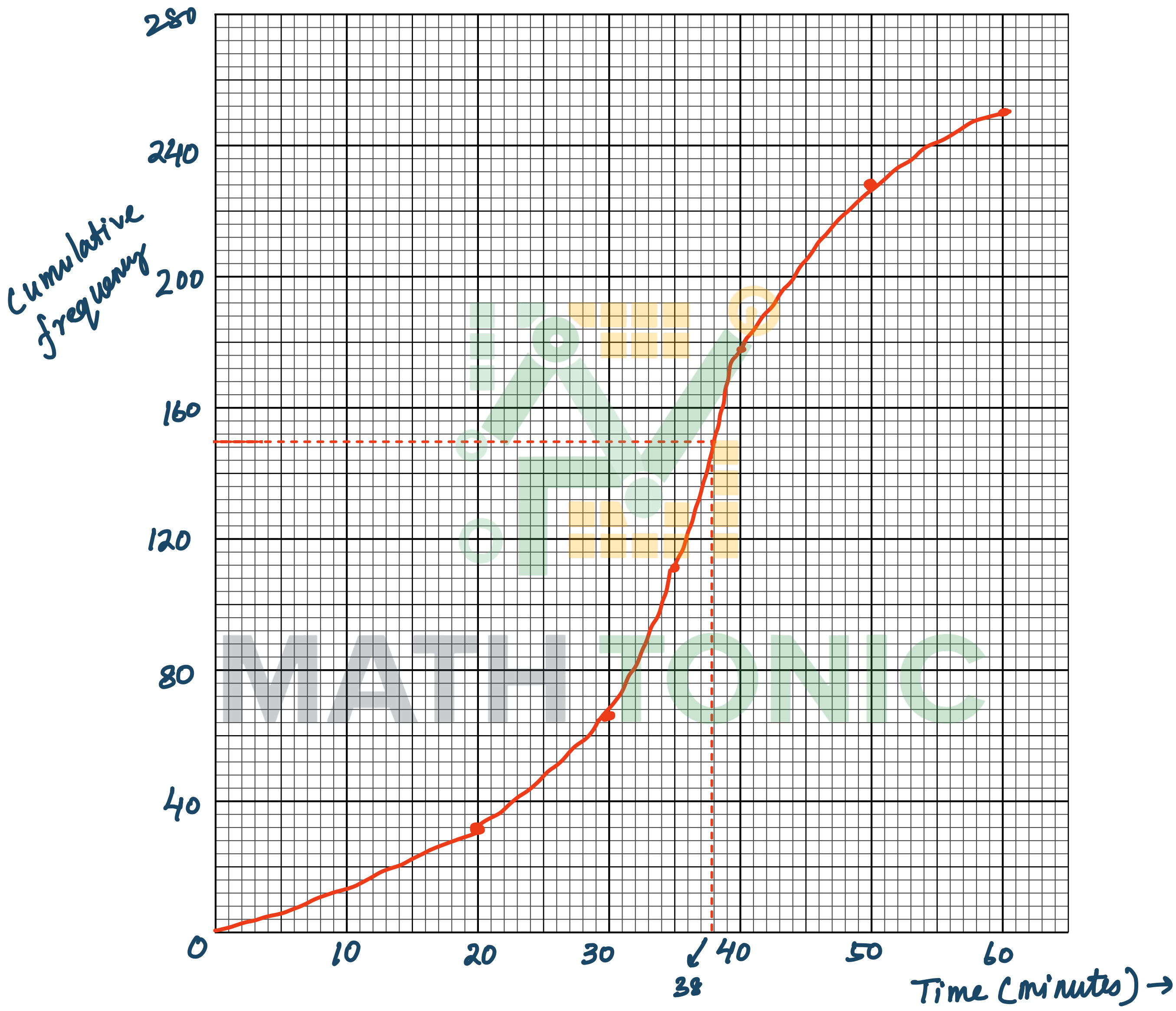
72.73

19.

The times, t minutes, taken to complete a walking challenge by 250 members of a club are summarised in the table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 60$
Cumulative frequency	32	66	112	178	228	250

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 60th percentile of the data. [1]

$\frac{60}{100} \times 250 = 150^{th}$

38 minutes

It is given that an estimate for the mean time taken to complete the challenge by these 250 members is 34.4 minutes.

(c) Calculate an estimate for the standard deviation of the times taken to complete the challenge by these 250 members. [4]

	0–20	20–30	30–35	35–40	40–50	50–60
Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 60$
Cumulative frequency	32	66	112	178	228	250
Midpoint (x)	10	25	32.5	37.5	45	55
Frequency	32	34	46	66	50	22

$$\text{Var} = \frac{(10^2 \times 32) + (25^2 \times 34) + (32.5^2 \times 46) + (37.5^2 \times 66) + (45^2 \times 50) + (55^2 \times 22)}{250} - (34.4)^2$$

Variance $= 151.24$

σ (Standard Deviation) : $\sqrt{151.24}$

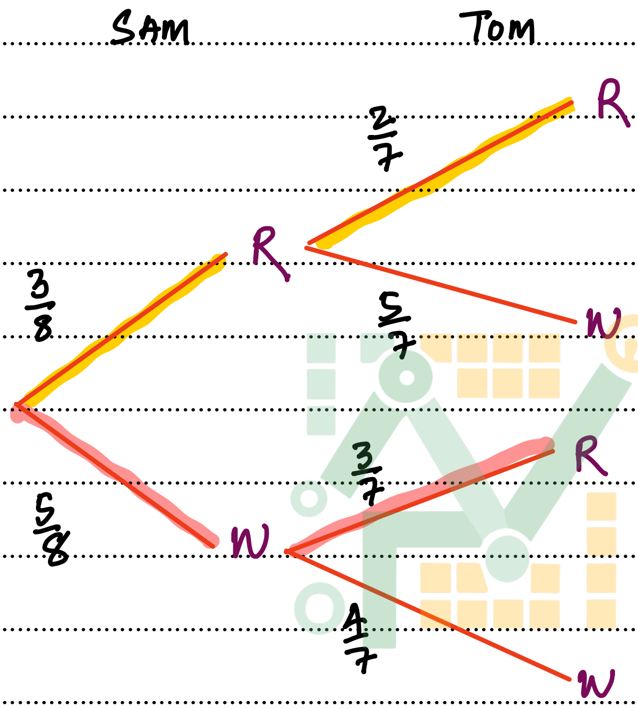
$= 12.3$

20.

Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game.

Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on his first turn. [2]



$$\left(\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{5}{8}\times\frac{3}{7}\right)$$

$$\frac{6}{56}+\frac{15}{56}$$

$$\frac{3}{8}$$

(b) Find the probability that Tom wins the game on his second turn. [4]

Probability: $P(R_S R_T W_S R_T) + P(W_S R_T R_S R_T)$

$+ P(W_S R_T W_S R_T)$

$\left(\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} \times \frac{1}{5}\right) + \left(\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5}\right)$

$\frac{30}{1680} + \frac{30}{1680} + \frac{120}{1680}$

$\frac{180}{1680} = \frac{3}{28}$

(c) Find the probability that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. [2]

Conditional probability

$P(R_S \mid T_{\text{wins 2nd term}}) = \dfrac{P(R_S R_T W_S R_T)}{3/28}$

$= \dfrac{30/1680}{3/28} = \dfrac{1}{6}$

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.