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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME: SOLVED BY MR. PABITRA

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2025**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 A curve has equation $y = 5 + 3x - 2x^2$ and a straight line has equation $y = kx + 13$, where k is a constant.

Find the set of values of k for which the curve and the line do **not** meet. [4]

$y = 5 + 3x - 2x^2$ $\qquad$ $y = kx + 13$

$5 + 3x - 2x^2 = kx + 13$

$2x^2 + kx - 3x + 13 - 5 = 0$

$2x^2 + (k-3)x + 8 = 0$

$a = 2 \qquad b = k - 3 \qquad c = 8$

Lines do not meet,

$b^2 - 4ac < 0$

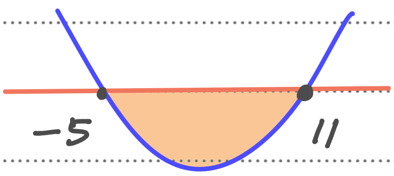
$(k-3)^2 - 4 \times 2 \times 8 < 0$

$(k-3)^2 - 64 < 0$

$k^2 - 6k + 9 - 64 < 0$

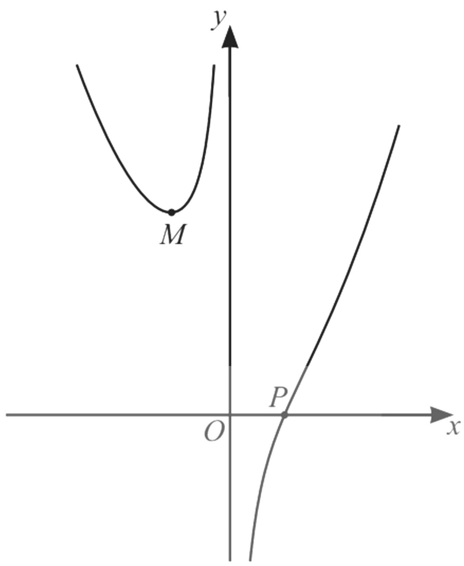
$k^2 - 6k - 55 < 0$

$(k-11)(k+5) < 0$ $\qquad$ $k = 11 \quad k = -5$



$\boxed{-5 < k < 11}$

2



The diagram shows the curve with equation $y = 2x^2 - \frac{5}{x} + 3$. The curve crosses the x-axis at the point $P(1, 0)$ and M is a minimum point.

(a) Find the gradient of the curve at P. [2]

$y = 2x^2 - 5x^{-1} + 3$

$\frac{dy}{dx} = 4x - 5(-1)x^{-2}$

$P(1,0)$

at $x = 1$

$\frac{dy}{dx} = 4x + \frac{5}{x^2}$

$\boxed{\text{gradient} = 9}$

$\frac{dy}{dx} = 4(1) + \frac{5}{1^2} = 9$

(b) Find the coordinates of M. Give each coordinate correct to 3 significant figures. [3]

At minimum point

$\frac{dy}{dx} = 0$

$4x + \frac{5}{x^2} = 0$

$4x^3 + 5 = 0$

$4x^3 = -5$

$x^3 = -\frac{5}{4}$

$x = \sqrt[3]{-\frac{5}{4}}$

$x = -1.08$

$y = 2x^2 - \frac{5}{x} + 3$

$y = 2(-1.08)^2 - \frac{5}{(-1.08)} + 3$

$y = 9.96$

$\boxed{x = -1.08 \quad y = 9.96}$

3 (a) Find the complete expansion of $\left(2x - \frac{3}{x}\right)^4$. [4]

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \ldots$

$\left(2x - \frac{3}{x}\right)^4 = (2x)^4 + {}^4C_1 (2x)^3 \left(\frac{-3}{x}\right)^1 + {}^4C_2 (2x)^2 \left(\frac{-3}{x}\right)^2 + {}^4C_3 (2x)^1 \left(\frac{-3}{x}\right)^3 + {}^4C_4 (2x)^0 \left(\frac{-3}{x}\right)^4$

$= 16x^4 + 4(8x^3)\left(\frac{-3}{x}\right) + 6(4x^2)\left(\frac{9}{x^2}\right) + 4(2x)\left(\frac{-27}{x^3}\right) + 1(1)\frac{81}{x^4}$

$= 16x^4 - 96x^2 + 216 - \frac{216}{x^3} + \frac{81}{x^4}$

(b) Hence determine the coefficient of x^2 in the expansion of $(x^2 + 5)\left(2x - \frac{3}{x}\right)^4$. [2]

$(x^2+5)\left(2x - \frac{3}{x}\right)^4$

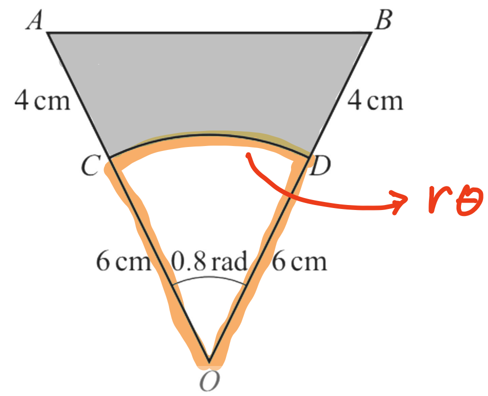
$(x^2+5)\left(16x^4 - 96x^2 + 216 - \frac{216}{x^3} + \frac{81}{x^4}\right)$

$216x^2 + 5(-96x^2)$

$216x^2 - 480x^2$

$-264x^2$

$\boxed{\text{Coefficient of } x^2 : -264}$

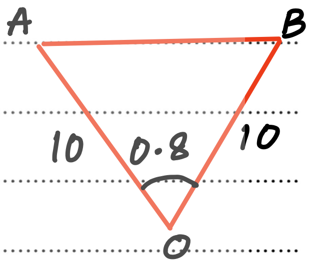


The diagram shows a triangle OAB where $OA = OB = 10$ cm and angle $AOB = 0.8$ radians. Points C and D on OA and OB respectively are such that the arc CD is part of a circle with centre O and radius 6 cm. The shaded region is bounded by the arc CD and the line segments CA, AB and BD.

(a) Find the perimeter of the shaded region. Cosine Rule [3]

$AB = \sqrt{10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 0.8}$

$AB = 7.788$



Arc length(S) = $r\theta$

Arc CD : $r\theta$

Arc CD $= 6 \times 0.8 = 4.8$ cm

Perimeter of Shaded Region:

Arc CD + AB + AC + BD

$4.8 + 7.788 + 4 + 4 = 20.6$

(b) Find the area of the shaded region. [3]

Area of shaded region:

Area of triangle OAB − Area of Sector OCD

$\frac{1}{2}ab\sin C$ $\qquad$ $\frac{1}{2}r^2\theta$

$\frac{1}{2} \times 10 \times 10 \times \sin(0.8) - \frac{1}{2} \times 6^2 \times 0.8$

$35.87 - 14.4$

$\boxed{21.47 \text{ cm}^2}$

5 An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400. [6]

$a = 5 \qquad d = 6$

$t_n = a + (n-1)d$

$150 = 5 + (n-1)6$

$145 = (n-1)6$

$\frac{145}{6} = n-1$

$n = \frac{145}{6} + 1$

$n = 25.16$

$t_n = a + (n-1)d$

$400 = 5 + (n-1)6$

$395 = (n-1)6$

$\frac{395}{6} = (n-1)$

$n = \frac{395}{6} + 1$

$n = 66.67$

Sum of all terms lie between 150 and 400

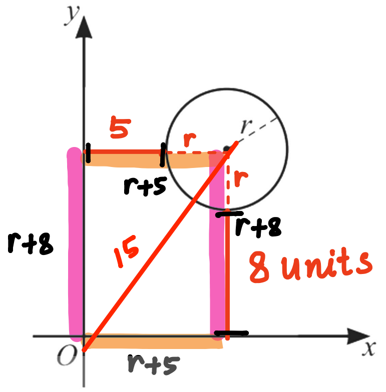
$S_{66} - S_{25}$

$\frac{66}{2}[2 \times 5 + (66-1)6] - \frac{25}{2}[2 \times 5 + (25-1)6]$

$13200 - 1925$

$\boxed{11275}$

6



The diagram shows a circle C of radius r, where $x > 0$ and $y > 0$ for all points on C. The least distance between any point on C and the x-axis is 8 units, and the least distance between any point on C and the y-axis is 5 units.

(a) State the coordinates of the centre of the circle in terms of r. [1]

$(r+5,\ r+8)$

(b) Given that the distance between the origin and the centre of the circle is 15 units, find the value of r. [3]

$(r+5)^2 + (r+8)^2 = 15^2$

$r^2 + 10r + 25 + r^2 + 16r + 64 = 15^2$

$2r^2 + 26r + 89 = 225$

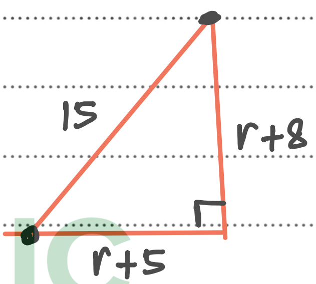
$2r^2 + 26r - 136 = 0$

$r^2 + 13r - 68 = 0$

$(r+17)(r-4) = 0$

$r = -17$ $\qquad r = 4$ $\qquad \boxed{r = 4}$

(rejected) radius can't be negative



(c) The point on the circle furthest from the origin is denoted by P.

Find the gradient of the tangent to the circle at P. [2]

Centre $(r+5, r+8)$

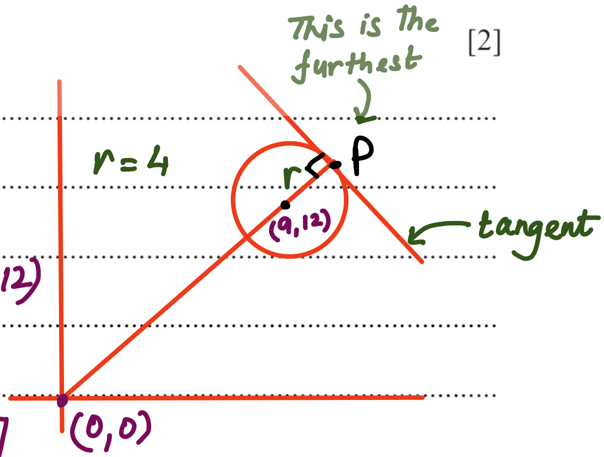
$(4+5,\ 4+8) = (9, 12)$

gradient of line passing through $(0,0)$ & $(9,12)$

$\dfrac{12-0}{9-0} = \dfrac{4}{3}$

tangent is perpendicular to line

$\boxed{m_{tangent} = -\dfrac{3}{4}}$ $\quad [m_1 \times m_2 = -1]$



7 **(a)** Show that $3\tan^2\theta + 5\sin^2\theta \equiv \dfrac{8\sin^2\theta - 5\sin^4\theta}{1-\sin^2\theta}$. [3]

$3\tan^2\theta + 5\sin^2\theta$

$3\dfrac{\sin^2\theta}{\cos^2\theta} + 5\sin^2\theta$

$\dfrac{3\sin^2\theta + 5\sin^2\theta\cos^2\theta}{\cos^2\theta}$

$\dfrac{3\sin^2\theta + 5\sin^2\theta(1-\sin^2\theta)}{1-\sin^2\theta}$

$\dfrac{3\sin^2\theta + 5\sin^2\theta - 5\sin^4\theta}{1-\sin^2\theta}$

$\dfrac{8\sin^2\theta - 5\sin^4\theta}{1-\sin^2\theta}$

(b) Hence solve the equation $3\tan^2\theta + 5\sin^2\theta = 9$ for $0° < \theta < 270°$. [4]

$$\frac{8\sin^2\theta - 5\sin^4\theta}{1-\sin^2\theta} = 9$$

$$8\sin^2\theta - 5\sin^4\theta = 9 - 9\sin^2\theta$$

$$5\sin^4\theta - 9\sin^2\theta - 8\sin^2\theta + 9 = 0$$

$$5\sin^4\theta - 17\sin^2\theta + 9 = 0$$

Let $\sin^2\theta = y$

$$5y^2 - 17y + 9 = 0$$

$$y = \frac{-(-17) \pm \sqrt{(-17)^2 - 4\times5\times9}}{2\times5}$$

$y = 2.744$

$\sin^2\theta = 2.744$

$\sin\theta = \sqrt{2.744}$

$\sin\theta = 1.69$

(REJECT) NOT POSSIBLE

$y = 0.655$

$\sin^2\theta = 0.655$

$\sin\theta = \pm\sqrt{0.655}$

$\sin\theta = \pm 0.809$

$\sin\theta = 0.809$

$\theta = \sin^{-1}(0.809)$

$\theta = 54.1, 125.9°$

$\sin\theta = -0.809$

$\theta = \sin^{-1}(-0.809)$

$\theta = 234.1°$

$$\boxed{\theta = 54.1°, 125.9°, 234.1°}$$

8 A geometric progression is such that its second term is -120 and its sum to infinity is 160.

(a) Find the common ratio. [4]

$t_n = ar^{n-1}$

$a, ar, ar^2, \ldots$

$t_2 = -120$

$ar = -120$

$a = \frac{-120}{r}$

$S_\infty = \frac{a}{1-r}$

$\boxed{\text{for } S_\infty, \ -1 < r < 1}$

$160 = \frac{a}{1-r}$

$160(1-r) = a$

$160(1-r) = \frac{-120}{r}$

$r(1-r) = \frac{-120}{160}$

$r - r^2 = -\frac{3}{4}$

$4r - 4r^2 = -3$

$4r^2 - 4r - 3 = 0$

$(2r-3)(2r+1) = 0$

$2r - 3 = 0 \qquad 2r + 1 = 0$

$r = \frac{3}{2}$ (reject) $\qquad \boxed{r = \frac{-1}{2}}$

(b) The first nine terms of the progression are now removed.

Find the sum to infinity of the remaining terms of the progression. [3]

Now 10th term is the first term

$a = \frac{-120}{r}$

$a = \frac{-120}{-1/2}$

$a = 240$

$t_{10} = ar^{10-1}$

$t_{10} = 240 \times \left(\frac{-1}{2}\right)^9$

$t_{10} = -\frac{240}{512}$

$a_{new} = \frac{-240}{512}$

$S_\infty = \frac{a}{1-r} = \frac{-240/512}{1-(-1/2)}$

$$\boxed{S_\infty = \frac{-5}{16} = -0.3125}$$

9 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{5}{x^3}$. It is given that the curve has a stationary point at $\left(\frac{1}{2}, 9\right)$.

(a) Use the expression for $\frac{d^2y}{dx^2}$ to determine whether the stationary point is a maximum or a minimum point. [2]

$$\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{5}{x^3}$$

at $x = \frac{1}{2}$ $\quad \frac{d^2y}{dx^2} = \frac{6}{(\frac{1}{2})^4} - \frac{5}{(\frac{1}{2})^3} = 56 \ (>0)$

Minimum point

(b) Find the equation of the curve. [7]

$$\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{5}{x^3}$$

$$\frac{dy}{dx} = \int (6x^{-4} - 5x^{-3})$$

$$\frac{dy}{dx} = \frac{6x^{-3}}{-3} - \frac{5x^{-2}}{-2} + c$$

$$\frac{dy}{dx} = -2x^{-3} + \frac{5}{2}x^{-2} + c$$

at $x = \frac{1}{2}$, $\frac{dy}{dx} = 0$ (stationary point)

$$0 = -2\left(\tfrac{1}{2}\right)^{-3} + \tfrac{5}{2}\left(\tfrac{1}{2}\right)^{-2} + c$$

$$0 = -16 + 10 + c$$

$$c = 6$$

$$\frac{dy}{dx} = -2x^{-3} + \frac{5}{2}x^{-2} + 6$$

$$\int dy = \int -2x^{-3} \cdot dx + \int \frac{5}{2}x^{-2} \cdot dx + \int 6 \cdot dx$$

$$y = -2\frac{x^{-2}}{-2} + \left(\frac{5}{2}\right)\frac{x^{-1}}{-1} + 6x + D$$

$$y = \frac{-2x^{-2}}{-2} + \left(\frac{5}{2}\right)\frac{x^{-1}}{-1} + 6x + D$$

at $\left(\frac{1}{2}, 9\right)$

$$9 = \frac{-2\left(\frac{1}{2}\right)^{-2}}{-2} - \frac{5}{2}\left(\frac{1}{2}\right)^{-1} + 6\left(\frac{1}{2}\right) + D$$

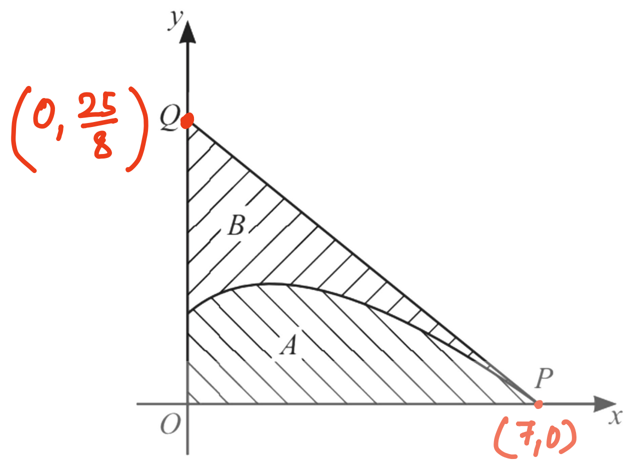
$$9 = \left(\frac{1}{2}\right)^{-2} - 5 + 3 + D$$

$$9 = 4 - 5 + 3 + D$$

$$D = 7$$

$$\boxed{y = \frac{1}{x^2} - \frac{5}{2x} + 6x + 7}$$

10



The diagram shows the curve with equation

$$y = 4(3x+4)^{\frac{1}{2}} - 2x - 6$$

for values of x such that $0 \leqslant x \leqslant 7$. The tangent to the curve at the point $P\,(7, 0)$ meets the y-axis at the point Q. Region A is bounded by the curve and the two axes. Region B is bounded by the curve, the line segment PQ and the y-axis.

(a) Find the area of region A. [4]

$$y = 4(3x+4)^{\frac{1}{2}} - 2x - 6$$

$$\text{Area} = \int_0^7 \left(4(3x+4)^{\frac{1}{2}} - 2x - 6\right) dx$$

$$= \left[\frac{4(3x+4)^{3/2}}{3 \times \frac{3}{2}} - \frac{2x^2}{2} - 6x\right]_0^7$$

$$= \left[\frac{8}{9}(3x+4)^{3/2} - x^2 - 6x\right]_0^7$$

$$= \left[\left(\frac{8}{9}[3\times7+4]^{3/2} - 7^2 - 6\times7\right) - \left(\frac{8}{9}\times4^{3/2}\right)\right]$$

$$= \frac{8}{9}\times25^{3/2} - 49 - 42 - \frac{64}{9}$$

$$= \boxed{13}$$ Square unit

(b) Find the area of region B. [5]

$y = 4(3x+4)^{\frac{1}{2}} - 2x - 6$

$\frac{dy}{dx} = 4 \times \frac{1}{2}(3x+4)^{-\frac{1}{2}} \times 3 - 2$

$\frac{dy}{dx} = 6(3x+4)^{-\frac{1}{2}} - 2$

at $P(7, 0)$

$x = 7$

$\frac{dy}{dx} = 6(3\times7+4)^{-\frac{1}{2}} - 2$

$\frac{dy}{dx} = 6 \times 25^{-\frac{1}{2}} - 2$

$\frac{dy}{dx} = \frac{6}{5} - 2 = \frac{-4}{5}$

gradient of line PQ

Equation of line PQ:

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-4}{5}(x - 7)$

$5y = -4(x-7)$

$5y = -4x + 28$

$5y + 4x = 28$

when $x = 0$

$5y + 0 = 28$

$y = \frac{28}{5}$

$Q\left(0, \frac{28}{5}\right)$

Area of region B:

$\text{Area}_{\Delta OPQ} - \text{Area}_{\text{region } A}$

$\frac{1}{2} \times 7 \times \frac{28}{5} - 13$

$19.6 - 13$

$\boxed{6.6 \text{ square unit.}}$

11 Functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - c \quad \text{and} \quad g(x) = 2x + k,$$

where c and k are positive constants. It is given that $g^{-1}(3k+1) = c$.

(a) Show that $gf(x) = 8x^2 - k - 1$. [4]

$g(x) = 2x + k$

$gf(x) = 2f(x) + k$

$= 2(4x^2 - c) + k$

$= 8x^2 - 2c + k$

$= 8x^2 - 2\left[\frac{2k+1}{2}\right] + k$

$= 8x^2 - (2k+1) + k$

$= 8x^2 - 2k - 1 + k$

$gf(x) = 8x^2 - k - 1$

let $y = g(x)$

$y = 2x + k$

$y - k = 2x$

$x = \frac{y-k}{2}$

$g^{-1}(x) = \frac{x-k}{2}$

$g^{-1}(3k+1) = \frac{3k+1-k}{2}$

$c = \frac{2k+1}{2}$

$2c = 2k+1$

$c = \frac{2k+1}{2}$

(b) The curve with equation $y = 8x^2 - k - 1$ is transformed to the curve with equation $y = \mathrm{h}(x)$ by the following sequence of transformations.

Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Stretch in the y-direction by scale factor k

Reflection in the x-axis

Find an expression for $\mathrm{h}(x)$ in terms of x and k. [3]

$f(x) = 8x^2 - k - 1$

Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$: $f(x-2)+3 = 8(x-2)^2 - k - 1 + 3$

$= 8(x-2)^2 - k + 2$

Stretch in y-direction by SF K :

$k[f(x-2)+3] = k[8(x-2)^2 - k + 2]$

Reflection in x-axis :

$-k[f(x-2)+3] = -k[8(x-2)^2 - k + 2]$

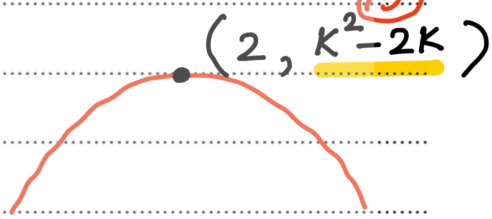
$h(x) = -8k(x-2)^2 + k^2 - 2k$

(c) The range of h is given by $\mathrm{h}(x) \leqslant 15$.

Find the values of c and k. [3]

$a(x-b)^2 + c$ Turning point (b, c)

$h(x) = \underbrace{-8k}_{a}(x-\underbrace{2}_{b})^2 + \underbrace{k^2 - 2k}_{c}$



$k^2 - 2k = 15$

$k^2 - 2k - 15 = 0$

$k = 5 \qquad k = -3$ (Reject)

$\boxed{k = 5}$

$c = \frac{1}{2}(2k+1)$

$c = \frac{1}{2}(10+1)$

$\boxed{c = \frac{11}{2}}$

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.