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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME: SOLVED BY MR. PABITRA

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **February/March 2025**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Jacob throws three coins at the same time.
The first coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$.
The second coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$.
The third coin is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{5}$.
The random variable X is the number of heads obtained.

$P(H_1) = \frac{1}{3}$, $P(T_1) = \frac{2}{3}$, $P(H_2) = \frac{1}{4}$, $P(T_2) = \frac{3}{4}$, $P(H_3) = \frac{1}{5}$, $P(T_3) = \frac{4}{5}$

(a) Show that $P(X = 2) = \frac{3}{20}$. [1]

2 Heads

$P(H_1\ H_2\ T_3) + P(H_1\ T_2\ H_3) + P(T_1, H_2, H_3)$

$\left(\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5}\right) + \left(\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5}\right)$

$\frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60} = \frac{3}{20}$

(b) Draw up the probability distribution table for X. [3]

X	0	1	2	3
$P(X=x)$	$2/5$	$13/30$	$3/20$	$1/60$

	①	②	③	
No head	T_1	T_2	T_3	$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{60} = 2/5$
One head	H_1	T_2	T_3	$\frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} = 12/60$
	T_1	H_2	T_3	$\frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} = 8/60$
	T_1	T_2	H_3	$\frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = 6/60$
Three head	H_1	H_2	H_3	$\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = 1/60$

$12/60 + 8/60 + 6/60$

$= 13/30$

(c) Given that $\mathrm{E}(X) = \frac{47}{60}$, find $\mathrm{Var}(X)$. [2]

x	0	1	2	3
$P(X=x)$	$2/5$	$13/30$	$3/20$	$1/60$

$$\mathrm{Var}(x) = \sum x^2 \cdot p(x) - [E(x)]^2$$

$$= \left[0^2 \times \frac{2}{5} + 1^2 \times \frac{13}{30} + 2^2 \times \frac{3}{20} + 3^2 \times \frac{1}{60}\right] - \left(\frac{47}{60}\right)^2$$

$$= \left(\frac{13}{30} + \frac{12}{20} + \frac{9}{60}\right) - \left(\frac{47}{60}\right)^2$$

$$= \frac{2051}{3600}$$

$$= 0.570$$

2 Last year, an online store sold a large number of computers. 55% of the computers were made by company F, 30% were made by company G and 15% were made by company H.

A random sample of 3 customers who each bought a computer from this store is chosen.

(a) Find the probability that the 3 customers bought computers all made by different companies. [1]

$P(F) = 0.55$ $\quad P(G) = 0.3$ $\quad P(H) = 0.15$

There are 6 combination

Ex. FGH
FHG
GFH
GHF
HGF
HFG

$(0.55 \times 0.3 \times 0.15) \times 3!$

0.1485

Binomial distribution (n is given)

A random sample of 12 customers who each bought a computer from this store is chosen.

(b) Find the probability that fewer than 10 of these customers bought a computer made by company F. [3]

$n = 12$ $\quad p = 0.55$ (success) $\quad q = 0.45$ (Failure)

$\boxed{P(x=r) = {}^{n}C_{r}\, p^{r} q^{n-r}}$

$P(x < 10) = 1 - P(x \geq 10)$

$= 1 - [P(x=10) + P(x=11) + P(x=12)]$

$= 1 - [{}^{12}C_{10}\, 0.55^{10}\, 0.45^{2} + {}^{12}C_{11}\, 0.55^{11}\, 0.45^{2} + {}^{12}C_{12}\, 0.55^{12}\, 0.45^{0}]$

$= 0.958$

A random sample of 140 customers who each bought a computer from this store is chosen.

(c) Use a suitable approximation to find the probability that more than 24 of these customers bought a computer made by company H. [5]

Binomial $\rightarrow$ Normal

$n = 140$ $\qquad$ $\mu = np = 140 \times 0.15 = 21$ (>5)

(mean)

$p = 0.15$

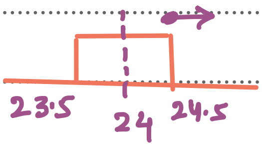
$q = 0.85$ $\qquad$ $\sigma = \sqrt{npq} = \sqrt{140 \times 0.15 \times 0.85}$

(S.D)

$= \sqrt{17.85}$

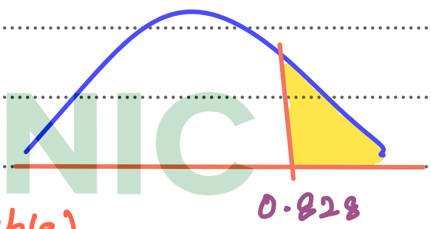
$P(x > 24)$ $\quad$ Continuity Correction required

$P(x > 24.5)$ $\quad$ $\frac{x-\mu}{\sigma}$



$P\left(z > \frac{24.5 - 21}{\sqrt{17.85}}\right)$

$P(z > 0.828)$



$1 - P(z < 0.828)$

(from z table)

$1 - 0.7961$

0.204

$\frac{1}{2} = 0.5$

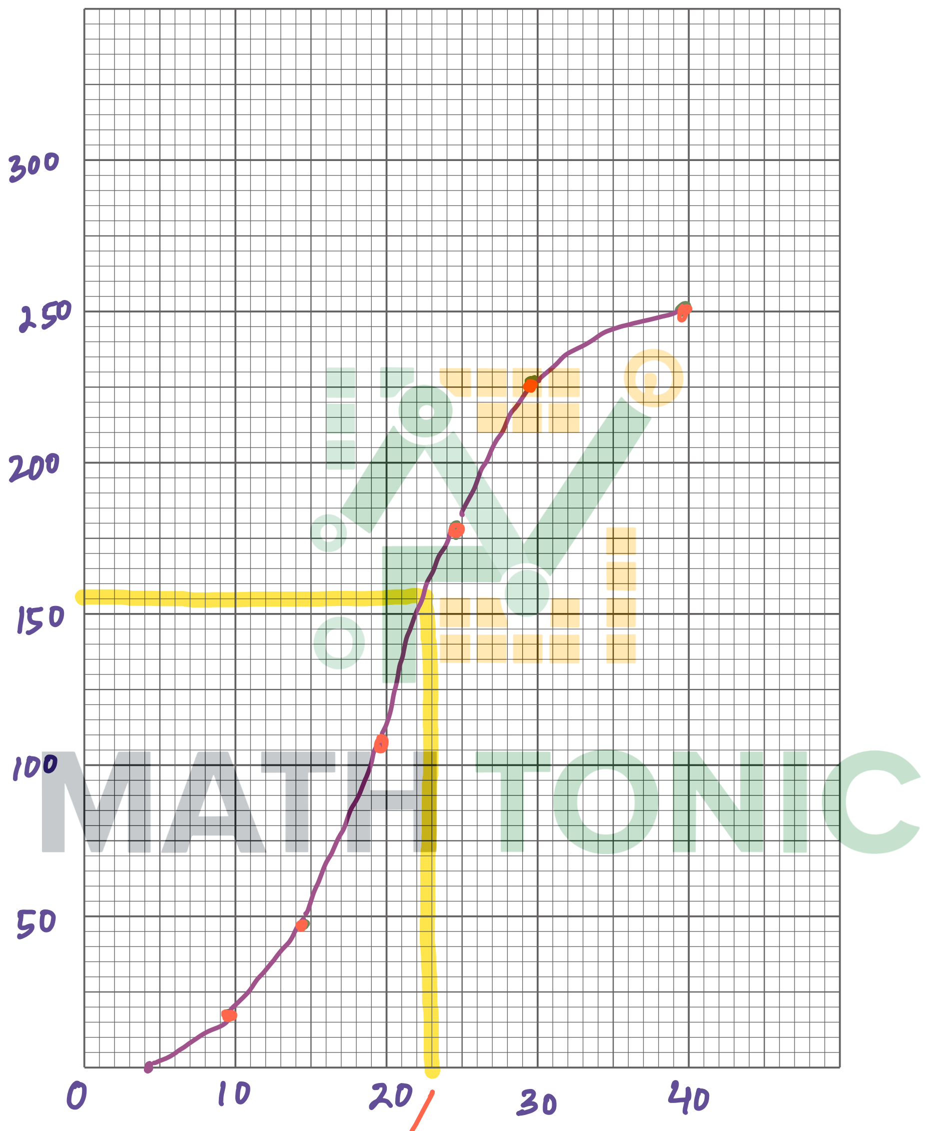
3 The lengths of 250 leaves of a certain type of plant are measured, correct to the nearest centimetre. The results are summarised in the table below.

1 cm

	4.5 9.5	9.5 14.5	14.5 19.5	19.5 24.5	24.5 29.5	29.5 39.5
Length (cm)	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 39
Frequency	18	28	60	72	48	24
cf	18	46	106	178	226	250

(9.5, 18) (14.5, 46)

(a) On the grid below, draw a cumulative frequency graph to illustrate this information. [4]



K = 23

DO NOT WRITE IN THIS MARGIN

(b) 38% of these leaves are of length k cm or more.

Use your graph to find an estimate for k. [2]

38% of $250 = \frac{38}{100} \times 250 = 95$

$250 - 95 = 155$

$k = 23$

(c) Calculate an estimate for the mean length of these 250 leaves. [3]

Class	mid value (x)	Freq (f)
4.5 − 9.5	7	18
9.5 − 14.5	12	28
14.5 − 19.5	17	60
19.5 − 24.5	22	72
24.5 − 29.5	27	48
29.5 − 39.5	34.5	24

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{(18\times 7)+(28\times 12)+(60\times 17)+(72\times 22)+(48\times 27)+(24\times 34.5)}{250}$$

$$= 20.76$$

4 Eddie has 16 toy cars, of which 8 are white, 5 are black and 3 are silver. He places all the cars in a bag and selects three of them at random, without replacement.

(a) Find the probability that all three cars are the same colour. [3]

$P(WWW) + P(BBB) + P(SSS)$

$\left(\frac{8}{16} \times \frac{7}{15} \times \frac{6}{14}\right) + \left(\frac{5}{16} \times \frac{4}{15} \times \frac{3}{14}\right) + \left(\frac{3}{16} \times \frac{2}{15} \times \frac{1}{14}\right)$

0.120

(b) Find the probability that, when the 3 cars are selected, at least one car is white and at least one car is black. [4]

W	B	S
1	1	1
2	1	0
1	2	0

$\frac{8}{16} \times \frac{5}{15} \times \frac{3}{14} \times 3!$ WBS

$\frac{8}{16} \times \frac{7}{15} \times \frac{5}{14} \times \frac{3!}{2!}$ WWB

$\frac{8}{16} \times \frac{5}{15} \times \frac{4}{14} \times \frac{3!}{2!}$ BBW

Addition:

0.607

5 The mass of peaches sold per day in a supermarket is normally distributed with mean 65.8 kg and standard deviation 9.6 kg.

(a) Find the probability that the mass of peaches sold on any given day is between 56 kg and 75 kg. [3]

$\mu = 65.8 \quad \sigma = 9.6$

$P(56 < x < 75)$

$P\left(\frac{56-65.8}{9.6} < z < \frac{75-65.8}{9.6}\right)$

$P(-1.02 < z < 0.9563)$

$P(a < z < b) = P(z < b) - P(z < a)$

$P(z < 0.9563) - P(z < -1.02)$

$P(z < -a) = 1 - P(z < a)$

$P(z < 0.9563) - [1 - P(z < 1.02)]$

$P(z < 0.9563) - 1 + P(z < 1.02)$

$0.8309 - 1 + 0.8463$

0.677

The mass of cherries sold per day in a supermarket is normally distributed with mean 72.4 kg and standard deviation σ kg. It is known that on 10% of days less than 59.1 kg of cherries are sold.

(b) Find the value of σ. [3]

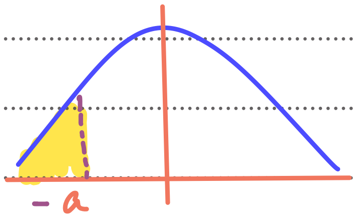
$\mu = 72.4 \quad \sigma = ?$

$P(x < 59.1) = 0.1$

$P\left(z < \frac{59.1 - 72.4}{\sigma}\right) = 0.1$

$P\left(z < \frac{-13.3}{\sigma}\right) = 0.1$ less than 0.5

must be negative



let $a = \frac{-13.3}{\sigma}$

$P(z < a) = 0.1$

Negative

$P(z < -a) = 0.1$

$1 - P(z < a) = 0.1$

$P(z < a) = 1 - 0.1 = 0.9$

$\phi^{-1}(0.9) = 1.282$

$a = 1.282$ (from table)

$a = -1.282$

$\frac{-13.3}{\sigma} = -1.282 \rightarrow \sigma = 10.4$

The supermarket is open 7 days a week.

(c) Find the probability that, in a randomly chosen week, the first day on which less than 59.1 kg of cherries are sold is the fifth day of the week. [1]

(Geometric Distribution)

$P = 0.1 \quad q = 0.9 \quad P(x=r) = q^{r-1} \cdot p$

$P(x=5) = 0.9^4 \times 0.1$

$= 0.065$

(d) Find the probability that, in a randomly chosen week, the first day on which less than 59.1 kg of cherries are sold is before the fifth day of the week. [2]

$P(x < 5) = P(x \le 4)$ must in geometric

$P(x \le r) = 1 - q^r$

$P(x \le 4) = 1 - 0.9^4$

$= 0.344$

6 Alissa has 10 different books from the series Squares and Circles. The books look similar except for their colour. There are 3 blue books, 2 red books, 2 yellow books, 1 orange book, 1 purple book and 1 green book.

Permutation

Alissa places the books in a row on her shelf. She is only interested in the arrangement of the colours.

(a) How many different colour arrangements are there of the 10 books? [1]

$$\frac{10!}{3! \times 2! \times 2!} = 151200$$

Blue Red Yellow

(b) How many different colour arrangements are there of the 10 books in which the 3 blue books are together, but the 2 yellow books are **not** next to each other? [2]

$\boxed{B \quad B \quad B}$ _ _ _ _ _ _ _

Three Blue books together : $\dfrac{8!}{2! \times 2!}$

Red Yellow

$\boxed{B \quad B \quad B}$ $\boxed{Y \quad Y}$ _ _ _ _ _

3 Blue and 2 Yellow together : $\dfrac{7!}{2!}$

Red

3 Blue and 2 Yellow Not together : $\dfrac{8!}{2! \times 2!} - \dfrac{7!}{2!}$

7560

(c) How many different colour arrangements are there of the 10 books with exactly 4 books between the 2 yellow books? [3]

1 Y ✓ ✓ ✓ ✓ Y ✓ ✓ ✓ ✓
2 _ Y _ _ _ _ Y _ _ _
3 _ _ Y _ _ _ _ Y _ _
4 _ _ _ Y _ _ _ _ Y _
5 _ _ _ _ Y _ _ _ _ Y

$$5 \times \frac{8!}{3! \times 2!} = 16800$$

Blue Red

Alissa selects 4 books from her 10 different books from the series Squares and Circles.

(d) Find the number of different selections if the 4 books include at least 1 red book, at most 1 blue book and exactly 1 yellow book. [4]

R	B	Y	Other	
1	1	1	1	${}^2C_1 \times {}^3C_1 \times {}^2C_1 \times {}^3C_1 = 36$
2	1	1	0	${}^2C_2 \times {}^3C_1 \times {}^2C_1 = 6$
1	0	1	2	${}^2C_1 \times 1 \times {}^2C_1 \times {}^3C_2 = 12$
2	0	1	1	${}^2C_2 \times 1 \times {}^2C_1 \times {}^3C_1 = 6$

Total = 60

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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