

**Cambridge International AS & A Level**CANDIDATE
NAME**SOLVED BY MR. PABITRA**CENTRE
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MATHEMATICSwww.mathtonic.com**9709/32**

Paper 3 Pure Mathematics 3

February/March 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

For online Class**Contact - +974 70473422****INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show **all** necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

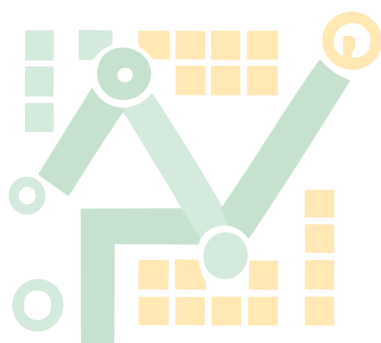
INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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MATH TONIC



1 Solve the equation

$$\ln(1 - e^{-2x}) + 3 = 0.$$

Give your final answer correct to 4 decimal places.

[3]

$$\ln(1 - e^{-2x}) + 3 = 0$$

$$\ln(1 - e^{-2x}) = -3$$

$$1 - e^{-2x} = e^{-3}$$

$$e^{-2x} = 1 - e^{-3}$$

$$-2x = \ln(1 - e^{-3})$$

$$x = -\frac{1}{2} \ln(1 - e^{-3})$$

$$x = 0.0255$$

$$\ln a = b$$

$$a = e^b$$

MATH TONIC



- 2 The equation of a curve is $xy^2 + \ln(x+2y) = 1$.

Find the gradient of the curve at the point where $x = 0$.

[5]

$$xy^2 + \ln(x+2y) = 1$$

When $x=0$,

$$\ln 2y = 1$$

$$2y = e^1$$

$$y = \frac{e}{2}$$

$$(0, \frac{e}{2})$$

$$xy^2 + \ln(x+2y) = 1$$

Implicit differentiation

$$x \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x) + \frac{d}{dx}[\ln(x+2y)] = \frac{d}{dx}(1)$$

$$x(2y) \frac{dy}{dx} + y^2 + \frac{1}{x+2y} \times \frac{d}{dx}[x+2y] = 0$$

$$2xy \cdot \frac{dy}{dx} + y^2 + \frac{1}{x+2y} \left[1 + 2 \frac{dy}{dx} \right] = 0$$

$$2xy \cdot \frac{dy}{dx} + y^2 + \frac{1}{x+2y} + \frac{2}{x+2y} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \left[2xy + \frac{2}{x+2y} \right] = -y^2 - \frac{1}{x+2y}$$

$$\frac{dy}{dx} = \frac{-y^2 - \frac{1}{x+2y}}{2xy + \frac{2}{x+2y}}$$

When $x=0$,

$$y = \frac{e}{2}$$

$$\frac{dy}{dx} = \frac{-y^2 - \frac{1}{2y}}{\frac{2}{2y}}$$



$$\frac{dy}{dx} = \frac{-\left(\frac{e}{2}\right)^2 - \frac{1}{2 \times e/2}}{2 \times e/2}$$

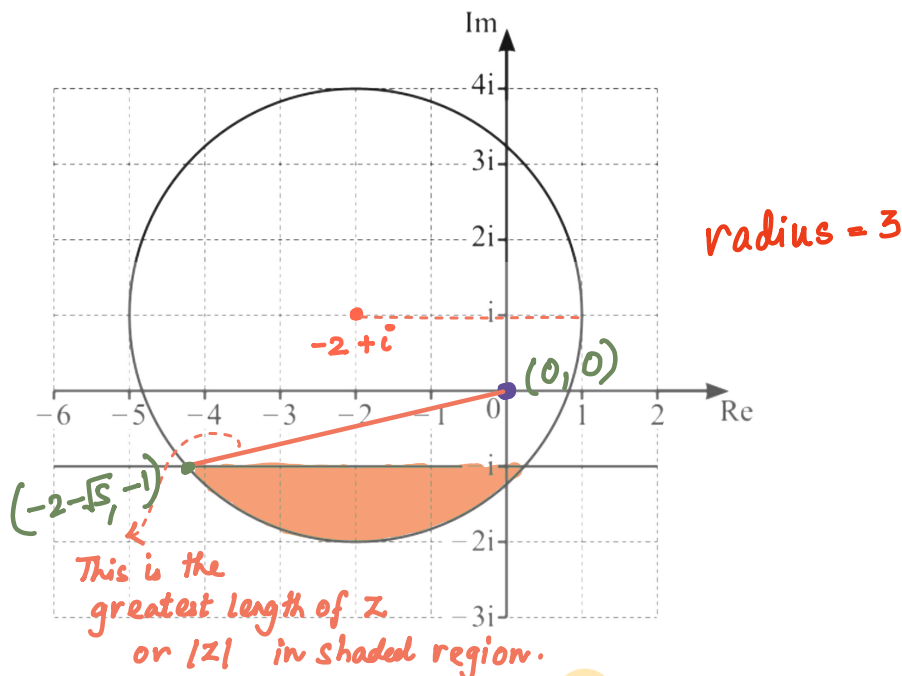
$$\frac{dy}{dx} = \frac{-\frac{e^2}{4} - \frac{1}{e}}{\frac{2}{e}}$$

$$\frac{dy}{dx} = \frac{e}{2} \left[-\frac{e^2}{4} - \frac{1}{e} \right]$$

$$\frac{dy}{dx} = -\frac{e^3}{8} - \frac{1}{2}$$

$$\frac{dy}{dx} = -3.01$$





The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the real axis. The boundaries of the region are included in the shaded region.

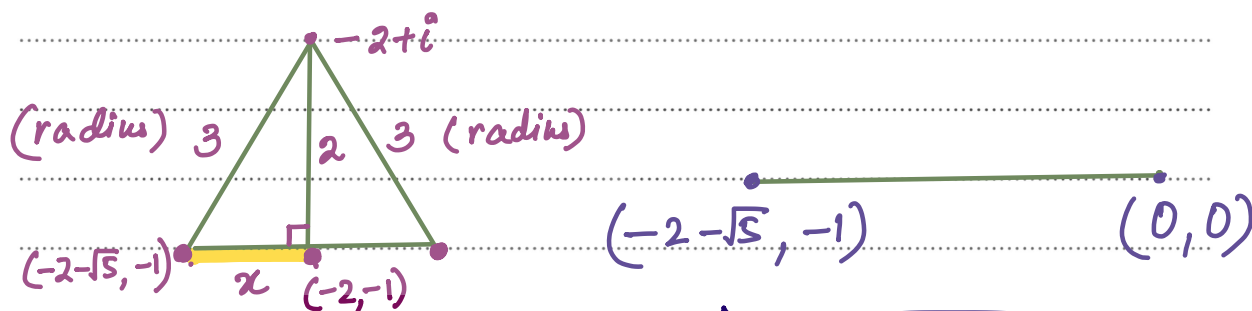
- (a) Find two inequalities in terms of z that define the shaded region. [3]

$$\text{Im}(z) \leq -1 \quad |z - (-2 + i)| \leq 3$$

$$\text{Im}(z) \leq -1 \quad \text{and} \quad |z + 2 - i| \leq 3$$

- (b) Find the greatest value of $|z|$ for points in this region. [3]

$$|z| = |z - (0 + 0i)| \quad \text{from origin } (0, 0)$$



$$x = \sqrt{3^2 - 2^2}$$

$$x = \sqrt{5}$$

$$|z| = \sqrt{(-2 - \sqrt{5} - 0)^2 + (-1 - 0)^2}$$

$$|z| = \sqrt{4 + 4\sqrt{5} + 5 + 1}$$

$$|z| = \sqrt{10 + 4\sqrt{5}} = 4.35$$

- 4 By first expressing the equation $\tan(x-60^\circ) = 2 \cot x$ as a quadratic equation in $\tan x$, solve the equation for $0^\circ \leq x \leq 180^\circ$. [6]

$$\tan(x-60^\circ) = 2 \cot x$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\frac{\tan x - \tan 60}{1 + \tan x \tan 60} = 2 \times \frac{1}{\tan x}$$

$$\tan x [\tan x - \tan 60] = 2 [1 + \tan x \tan 60]$$

$$\tan x [\tan x - \sqrt{3}] = 2 [1 + \sqrt{3} \tan x]$$

$$\tan^2 x - \sqrt{3} \tan x = 2 + 2\sqrt{3} \tan x$$

$$\tan^2 x - \sqrt{3} \tan x - 2 - 2\sqrt{3} \tan x = 0$$

$$\tan^2 x - 3\sqrt{3} \tan x - 2 = 0$$

$$a = 1 \quad b = -3\sqrt{3} \quad c = -2$$

$$\tan x = \frac{-(3\sqrt{3}) \pm \sqrt{(-3\sqrt{3})^2 - 4(1)(-2)}}{2 \times 1}$$

$$\tan x = \frac{3\sqrt{3} \pm \sqrt{27+8}}{2}$$

$$\tan x = -0.3599$$

$$\tan x = 5.5561$$

$$x = \tan^{-1}(-0.3599)$$

$$x = \tan^{-1}(5.5561)$$

$$x = -19.396$$

$$x = 79.8^\circ$$

$$x = 180 - 19.396$$

$$x = 160.2^\circ$$



- 5 The square roots of $-4+6\sqrt{5}i$ can be expressed in the Cartesian form $x+iy$, where x and y are real and exact.

By first forming a quartic equation in x or y , find the square roots of $-4+6\sqrt{5}i$ in exact Cartesian form. [5]

$$\sqrt{-4+6\sqrt{5}i} = x+iy$$

$$-4+6\sqrt{5}i = (x+iy)^2$$

$$-4+6\sqrt{5}i = x^2 + 2xyi + i^2y^2 \quad i^2 = -1$$

$$-4+6\sqrt{5}i = x^2 + 2xyi - y^2$$

$$-4+6\sqrt{5}i = (x^2-y^2) + 2xyi$$

$$x^2 - y^2 = -4$$

$$2xy = 6\sqrt{5}$$

$$x = \frac{6\sqrt{5}}{2y}$$

$$\left(\frac{3\sqrt{5}}{y}\right)^2 - y^2 = -4$$

$$x = \frac{3\sqrt{5}}{y}$$

$$\frac{45}{y^2} - y^2 = -4$$

$$45 - y^4 = -4y^2$$

$$y^4 - 4y^2 - 45 = 0$$

let $y^2 = m$

$$m^2 - 4m - 45 = 0$$

$$(m-9)(m+5) = 0$$

$$m = 9$$

$$m = -5$$

$$y^2 = 9$$

$$y^2 = -5$$

$$y = \pm 3$$

$y = \sqrt{-5}$ (Reject because $y = \text{real number}$)

$$y = 3$$

$$y = -3$$

$$x = \frac{3\sqrt{5}}{3}$$

$$x = \frac{3\sqrt{5}}{-3}$$

$$x = \sqrt{5}$$

$$x = -\sqrt{5}$$

$$\sqrt{5} + 3i$$

or

$$-\sqrt{5} - 3i$$



- 6 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = \left(\frac{1}{5}x + 1\right) \sin^2 2\theta,$$

and $x = 5$ when $\theta = 0$.

Solve the differential equation and obtain an expression for x in terms of θ .

[7]

$$\frac{dx}{d\theta} = \left(\frac{1}{5}x + 1\right) \sin^2 2\theta$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\sin^2 2\theta = \frac{1 - \cos 4\theta}{2}$$

$$\int \frac{dx}{\frac{1}{5}x + 1} = \int \sin^2 2\theta \cdot d\theta$$

$$\int \frac{1}{ax+b} \cdot dx = \frac{\ln(ax+b)}{a}$$

$$\frac{\ln\left(\frac{1}{5}x + 1\right)}{\frac{1}{5}} = \frac{1}{2} \int (1 - \cos 4\theta) d\theta$$

$$5 \ln\left(\frac{1}{5}x + 1\right) = \frac{1}{2} \left(\theta - \frac{\sin 4\theta}{4}\right) + C$$

$$\ln\left(\frac{1}{5}x + 1\right)^5 = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + C$$

when $x=5, \theta=0$

$$\ln\left(\frac{1}{5} \times 5 + 1\right)^5 = \frac{1}{2} \times 0 - \frac{1}{8} \sin 0 + C$$

$$\ln 2^5 = C$$

$$C = \ln 32$$

$$\ln\left(\frac{1}{5}x + 1\right)^5 = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 32$$

$$\left(\frac{1}{5}x + 1\right)^5 = e^{\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 32}$$

$$\left[\left(\frac{1}{5}x + 1 \right)^5 \right]^{1/5} = \left[e^{\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 32} \right]^{1/5}$$

$$\frac{1}{5}x + 1 = e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta + \frac{1}{5}\ln 32}$$

$$\frac{1}{5}x + 1 = e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta} \times e^{\frac{1}{5}\ln 32}$$

$$\frac{1}{5}x = e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta} \times e^{\ln 32^{1/5}} - 1$$

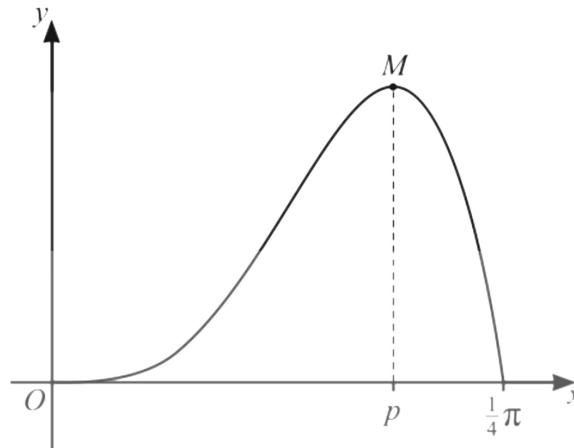
$e^{\ln a} = a$

$$\frac{1}{5}x = e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta} \times 32^{1/5} - 1$$

$$\frac{1}{5}x = e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta} \times 2 - 1$$

$$\frac{1}{5}x = 2e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta} - 1$$

$$x = 10e^{\frac{1}{10}\theta - \frac{1}{40}\sin 4\theta} - 5$$



The diagram shows the curve $y = x^3 \cos 2x$ for $0 \leq x \leq \frac{1}{4}\pi$. The curve has a maximum point at M , where $x = p$.

- (a) Show that p satisfies the equation $p = \frac{1}{2} \tan^{-1}\left(\frac{3}{2p}\right)$. [3]

$$y = x^3 \cos 2x$$

$$\frac{dy}{dx} = x^3 \frac{d}{dx}[\cos 2x] + \cos 2x \frac{d}{dx}(x^3)$$

$$\frac{dy}{dx} = -2x^3 \sin 2x + 3x^2 \cos 2x$$

(turning point)
at max. $\frac{dy}{dx} = x^2 [3 \cos 2x - 2x \sin 2x]$

$$\frac{dy}{dx} = 0 \quad 0 = x^2 [3 \cos 2x - 2x \sin 2x]$$

$$3 \cos 2x - 2x \sin 2x = 0$$

$$x^2 = 0$$

$$3 \cos 2x = 2x \sin 2x$$

(Reject)

$$3 = 2x \frac{\sin 2x}{\cos 2x}$$

$$3 = 2x \tan 2x$$

at $x = p$

$$3 = 2p \tan 2p$$

$$\tan 2p = \frac{3}{2p}$$

$$2p = \tan^{-1}\left(\frac{3}{2p}\right)$$

$$p = \frac{1}{2} \tan^{-1}\left(\frac{3}{2p}\right)$$

(b) Show by calculation that $0.5 < p < 0.7$.

[2]

$$p = \frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right)$$

$$p - \frac{1}{2} \tan^{-1} \left(\frac{3}{2p} \right)$$

when $p = 0.5$

$$0.5 - \frac{1}{2} \tan^{-1} \left(\frac{3}{2 \times 0.5} \right) = -0.1245$$

when $p = 0.7$

$$0.7 - \frac{1}{2} \tan^{-1} \left(\frac{3}{2 \times 0.7} \right) = +0.1329$$

Since there is a Sign change, root lies between 0.5 and 0.7

(c) Use an iterative formula based on the equation in part (a) to calculate p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Since $0.5 < p < 0.7$

take any value of p after 0.5

$$p_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{3}{2p_n} \right)$$

$$p_1 = 0.6$$

$$p_2 = \frac{1}{2} \tan^{-1} \left(\frac{3}{2 \times 0.6} \right) = 0.59514$$

$$p_3 = \frac{1}{2} \tan^{-1} \left(\frac{3}{2 \times 0.59514} \right) = 0.59654$$

$$p_4 = \frac{1}{2} \tan^{-1} \left(\frac{3}{2 \times 0.59654} \right) = 0.59614$$

$$p_5 = \frac{1}{2} \tan^{-1} \left(\frac{3}{2 \times 0.59614} \right) = 0.59626$$

$$p = 0.596$$

- 8 Two lines have equations $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$.

(a) Show that the lines are skew.

[5]

line 1 *line 2*

$$\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} -1 + 2\lambda \\ 3 + 3\lambda \\ -4 - \lambda \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 2 - \mu \\ -3 - 2\mu \\ -1 + \mu \end{pmatrix}$$

$$\begin{pmatrix} -1 + 2\lambda \\ 3 + 3\lambda \\ -4 - \lambda \end{pmatrix} = \begin{pmatrix} 2 - \mu \\ -3 - 2\mu \\ -1 + \mu \end{pmatrix}$$

$$-1 + 2\lambda = 2 - \mu \quad \text{--- (i)}$$

$$3 + 3\lambda = -3 - 2\mu \quad \text{--- (ii)}$$

$$\textcircled{i} \times 2 \Rightarrow -2 + 4\lambda = 4 - 2\mu$$

$$\textcircled{ii} \Rightarrow \underline{\underline{-3 + 3\lambda = -3 - 2\mu}}$$

$$-5 + \lambda = 7$$

$$\underline{\underline{\lambda = 12}}$$

$$-1 + 2\lambda = 2 - \mu$$

$$-1 + 2(12) = 2 - \mu$$

$$-1 + 24 = 2 - \mu$$

$$\underline{\underline{\mu = -21}}$$

Check in third equation:

$$-4 - \lambda = -1 + \mu$$

$$-4 - 12 = -1 + (-21)$$

$$-16 \neq -22$$

Since z coordinate did not satisfy the condition of intersection, the two lines are skew.



(b) Find the obtuse angle between the directions of the two lines.

[3]

$$d_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad d_2 = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

direction vector

$$a \cdot b = |a| |b| \cos \theta$$

$$\theta = \cos^{-1} \left[\frac{a \cdot b}{|a| |b|} \right]$$

$$\theta = \cos^{-1} \left[\frac{\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}}{\sqrt{2^2 + 3^2 + (-1)^2} \sqrt{(-1)^2 + (-2)^2 + 1^2}} \right]$$

$$\theta = \cos^{-1} \left[\frac{2(-1) + 3(-2) + (-1)(1)}{\sqrt{14} \sqrt{6}} \right]$$

$$\theta = \cos^{-1} \left[\frac{-9}{\sqrt{14} \sqrt{6}} \right]$$

$$\theta = 169.1^\circ$$



- 9 The polynomial $6x^3 + ax^2 + bx + 9$ is denoted by $p(x)$, where a and b are constants. It is given that $(x-3)$ is a factor of $p(x)$, and when the first derivative $p'(x)$ is divided by $(x-3)$ the remainder is 72.

(a) Find the values of a and b .

[5]

$(x-3)$ is a factor of $P(x)$, it means $P(3)=0$

$$P(x) = 6x^3 + ax^2 + bx + 9$$

$$P(3) = 6(3)^3 + a(3)^2 + b(3) + 9$$

$$0 = 162 + 9a + 3b + 9$$

$$9a + 3b = -171$$

$$3a + b = -57 \quad \text{--- (i)}$$

$$P(x) = 6x^3 + ax^2 + bx + 9$$

$$P'(x) = 18x^2 + 2ax + b$$

when $P'(x)$ is divided by $(x-3)$, remainder is 72

$$P'(3) = 72$$

$$P'(3) = 18(3)^2 + 2a(3) + b$$

$$72 = 162 + 6a + b$$

$$6a + b = -90 \quad \text{--- (ii)}$$

Solve Simultaneous equations

$$3a + b = -57$$

$$6a + b = -90$$

$$\underline{-3a} \quad = 33$$

$$a = -11$$

$$3a + b = -57$$

$$3(-11) + b = -57$$

$$b = -57 + 33$$

$$b = -24$$

$$a = -11 \quad b = -24$$



(b) When a and b have the values found in part (a), factorise $p(x)$ completely.

[3]

$$a = -11 \quad b = -24$$

$$\text{then } p(x) = 6x^3 - 11x^2 - 24x + 9$$

$$\begin{array}{r} x-3 \overline{) 6x^3 - 11x^2 - 24x + 9} \\ \underline{6x^3 + 7x - 3} \\ -11x^2 - 24x + 9 \\ \underline{+ 18x^2} \\ 7x^2 - 24x + 9 \\ \underline{- 7x^2 - 21x} \\ -3x + 9 \\ \underline{+ 3x + 9} \\ 0 \end{array}$$

$$6x^3 - 11x^2 - 24x + 9 = (x-3)(6x^2 + 7x - 3)$$

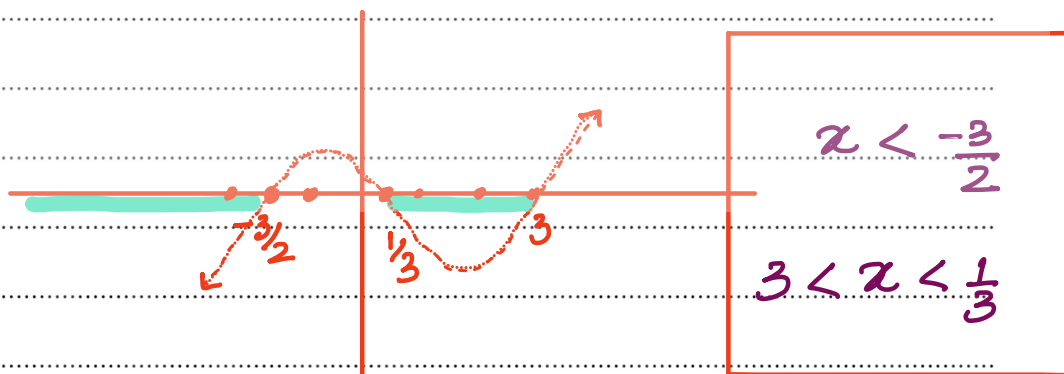
$$6x^3 - 11x^2 - 24x + 9 = (x-3)(2x+3)(3x-1)$$

(c) Hence solve the inequality $p(x) < 0$.

[2]

$$p(x) = (x-3)(2x+3)(3x-1)$$

$$x = 3 \quad x = -\frac{3}{2} \quad x = \frac{1}{3}$$



10 Let $f(x) = \frac{-7x^2 + 2x - 6}{(1+x)(4+x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

$$\frac{-7x^2 + 2x - 6}{(1+x)(4+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{4+x^2}$$

$$-7x^2 + 2x - 6 = A(4+x^2) + (Bx+C)(1+x)$$

let $x = -1$

$$-7(-1)^2 + 2(-1) - 6 = A(4+(-1)^2)$$

$$-15 = 5A$$

$$A = -3$$

let $x = 0$

$$-6 = 4A + C$$

$$-6 = 4(-3) + C$$

$$C = 6$$

let $x = 1$

$$-7 + 2 - 6 = 5A + 2(B+C)$$

$$-11 = 5(-3) + 2B + 2(6)$$

$$-11 = -15 + 2B + 12$$

$$2B = -8$$

$$B = -4$$

$$\frac{-7x^2 + 2x - 6}{(1+x)(4+x^2)} = \frac{-3}{1+x} + \frac{-4x+6}{4+x^2}$$

- (b) Hence find the exact value of $\int_0^2 f(x) dx$. Give your answer in the form $a\pi - \ln b$, where a and b are constants. [6]

$$\int_0^2 \left(\frac{6-4x}{4+x^2} - \frac{3}{1+x} \right) dx$$

$$\int_0^2 \frac{6}{4+x^2} dx - \int_0^2 \frac{4x}{4+x^2} dx - \int_0^2 \frac{3}{1+x} dx$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{u'}{u} du = \ln u + C$$

$$\left[6 \times \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) \right]_0^2 - \left[2 \ln(4+x^2) \right]_0^2 - \left[3 \ln(1+x) \right]_0^2$$

$$3 \left[\tan^{-1} \left(\frac{x}{2} \right) \right]_0^2 - 2 \left[\ln(4+x^2) \right]_0^2 - 3 \left[\ln(1+x) \right]_0^2$$

$$3 \left[\tan^{-1} \left(\frac{2}{2} \right) - \tan^{-1}(0) \right] - 2 \left[\ln(4+2^2) - \ln(4+0) \right] - 3 \left[\ln(1+2) - \ln(1) \right]$$

$\ln 1 = 0$

$$3 \tan^{-1}(1) - 2 \ln 8 + 2 \ln 4 - 3 \ln 3$$

$$\frac{3\pi}{4} - \ln 8^2 + \ln 4^2 - \ln 3^3$$

$$\frac{3\pi}{4} - \ln 64 + \ln 16 - \ln 27$$

$$\frac{3\pi}{4} + \ln 108^{-1}$$

$$\frac{3\pi}{4} + \ln 16 - (\ln 64 + \ln 27)$$

$$\frac{3\pi}{4} - \ln 108$$

$$\frac{3\pi}{4} + \ln 16 - \ln(64 \times 27)$$

$$a = \frac{3}{4}, b = 108$$

$$\frac{3\pi}{4} + \ln \left[\frac{16}{64 \times 27} \right] = \frac{3\pi}{4} + \ln \left(\frac{1}{108} \right)$$



- 11 Find the exact value of $\int_0^{\pi} x^2 \cos \frac{1}{3}x dx$.

[6]

$$\int_0^{\pi} x^2 \cos \frac{1}{3}x dx$$

I L A T E
 $\downarrow \quad \downarrow$
 $u \quad v$

let $u = x^2$

$$\frac{du}{dx} = 2x$$

$$dx$$

$$du = 2x dx$$

$$\int dv = \int \cos \frac{1}{3}x dx$$

$$v = \frac{\sin \frac{1}{3}x}{\frac{1}{3}} = 3 \sin \frac{1}{3}x$$

$$uv - \int v du$$

$$= 3x^2 \sin \frac{1}{3}x - \int 3 \sin \frac{1}{3}x \cdot 2x dx$$

$$= 3x^2 \sin \frac{1}{3}x - 6 \int x \cdot \sin \frac{1}{3}x dx$$

$$u = x$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\int dv = \int \sin \frac{1}{3}x dx$$

$$v = \frac{-\cos \frac{1}{3}x}{\frac{1}{3}} = -3 \cos \frac{1}{3}x$$

$$uv - \int v du$$

$$-3x \cos \frac{1}{3}x + \int 3 \cos \frac{1}{3}x dx$$

$$-3x \cos \frac{1}{3}x + 3 \int \cos \frac{1}{3}x dx$$



$$-3x \cos \frac{1}{3}x + 3x \frac{\sin \frac{1}{3}x}{\frac{1}{3}}$$

$$-3x \cos \frac{1}{3}x + 9 \sin \frac{1}{3}x$$

$$3x^2 \sin \frac{1}{3}x - 6 \left[-3x \cos \frac{1}{3}x + 9 \sin \frac{1}{3}x \right]$$

$$\left[3x^2 \sin \frac{1}{3}x + 18x \cos \frac{1}{3}x - 54 \sin \frac{1}{3}x \right]_{\pi}^0$$

$$\left(3\pi^2 \sin \frac{1}{3}\pi + 18\pi \cos \frac{1}{3}\pi - 54 \sin \frac{1}{3}\pi \right)$$

$$- (0 + 0 - 54 \sin 0)$$

$$3\pi^2 \times \frac{\sqrt{3}}{2} + 18\pi \times \frac{1}{2} - 54 \times \frac{\sqrt{3}}{2}$$

$$\frac{3\sqrt{3}\pi^2 + 9\pi - 27\sqrt{3}}{2}$$



If you use the following page to complete the answer to any question, the question number must be clearly shown.



MATH TONIC

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